Assignment 7 Solutions

Question 1

Let $\mathbf{M}$ be the mass of the empty rocket.
Let $\mathbf{M_A}$ be the mass of fuel A.
Let $\mathbf{M_B}$ be the mass of fuel B.

For type A fuel:

$$\mathbf{X} = 0 + \mathbf{v_e}\,\mathbf{ln}\left(\frac{\mathbf{M}+\mathbf{M_A}}{\mathbf{M}}\right) = \mathbf{v_e}\,\mathbf{ln}\left(\frac{2\mathbf{M}}{\mathbf{M}}\right) = \mathbf{v_e}\,\mathbf{ln}(2) \qquad (1)$$

For type B fuel:

$$\mathbf{X} = 0 + 3\,\mathbf{v_e}\,\mathbf{ln}\left(\frac{\mathbf{M}+\mathbf{M_B}}{\mathbf{M}}\right) \qquad (2)$$

Equating the right hand sides of (1) and (2) gives

$$3\,\mathbf{v_e}\,\mathbf{ln}\left(\frac{\mathbf{M}+\mathbf{M_B}}{\mathbf{M}}\right) = \mathbf{v_e}\,\mathbf{ln}(2)$$

$$\mathbf{ln}\left(\frac{\mathbf{M}+\mathbf{M_B}}{\mathbf{M}}\right) = \frac{\mathbf{ln}(2)}{3}$$

$\mathbf{ln}\left(\frac{\mathbf{M}+\mathbf{M_B}}{\mathbf{M}}\right) = 0.231$ and raising $\mathbf{e}$ to both sides of this equation gives:

$$\frac{\mathbf{M}+\mathbf{M_B}}{\mathbf{M}} = \mathbf{e}^{0.231}$$

$$1 + \frac{\mathbf{M_B}}{\mathbf{M}} = 1.26$$

$$\frac{\mathbf{M_B}}{\mathbf{M}} = 0.26$$

9-24

The problem can be solved in two steps: the collision (1), and the pendulum moving up (2).

Step 1: The collision. We don't know if the collision is ellastic or not. We can use conservation of momentum for the bullet (b) and pendulum (p) system:

$$p_{bi} + p_{pi} = p_{bf} + p_{pf}$$

$$mv_{bi} + Mv_{pi} = mv_{bf} + Mv_{pf}$$

Since we are told what is the final speed of the bullet $v_{bf} = v_{bi}/2$, we can substitute it in the equation:

$$mv_{bi} + 0 = mv_{bi}/2 + Mv_{pf}$$

We can isolate the speed of the pendulum after the collision:

$$v_{pf} = \frac{mv_{bi}}{2M}$$

Step 2: The pendulum moves up. For this stage, the pendulum acquired a speed v_{pf} from the collision, and it moves up. We have conservation of energy.

$$KE_i + PE_i = KE_f + PE_f$$

We can define the bottom as the reference height. The minimum speed is such that the pendulum will barely make it to the top ($h = 2l$), having lost all it's kinetic energy.

$$\frac{1}{2}Mv_{pf}^2 + 0 = 0 + Mg2l$$

Replace the equation for v_{pf}:

$$\frac{1}{2}M(\frac{mv_{bi}}{2M})^2 = Mg2l$$

Solve for v_{bi}:

$$v_{bi} = \frac{4M}{m}\sqrt{gl}$$

9-60

a) We have conservation of momentum of the block (b) and wedge (w) system.

$$p_{bi} + p_{wi} = p_{bf} + p_{wf}$$

$$0 + 0 = m_2 v_{wf} + m_1 v_{bf}$$

$$0 = 3.00\,kg\,v_{wf} + 0.500\,kg \times\ 4.00\,m/s$$

$$v_{wf} = -0.667\,m/s$$

b) We can use the conservation of momentum of the block (b) and wedge (w) system.

$$PE_{bi} + PE_{wi} + KE_{bi} + KE_{wi} = PE_{bf} + PE_{wf} + KE_{bf} + KE_{wf}$$

The wedge's potential energy is constant, and the initial kinetic energy of each is 0.

$$m_1 gh + 0 + 0 + 0 = 0 + 0 + \frac{1}{2} m_1 v_{bf}^2 + \frac{1}{2} m_2 v_{wf}^2$$

$$0.500\,kg\,9.8 m/s^2\,h + 0 + 0 + 0 = 0 + 0 + \frac{1}{2} 0.500 kg\,(4.00\,m/s)^2\, + \frac{1}{2} 3.00\,kg(-0.667\,m/s)^2$$

Isolate h:

$$h = 0.952 m$$

9-66

We can just use conservation of energy to find the speed of the ice cube when it leaves the ramp:

$$mgh_i = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.8\,m/s^2)\,1.5\,m} = 5.42\,m/s$$

The x component of the speed is:

$$v_x = 5.42\,m/s\,cos(40) = 4.15\,m/s$$

which remains constant trough the trajectory. When the ice cube reaches the top of its trajectory,

$$v_y = 0\,m/s$$

So we have:

$$v_{top} = 4.15\,m/s$$

During the collision with the wall, the impulse is:

$$I = mv_f - mv_i$$

$$I = 0.005\,kg(\frac{4.15\,m/s}{2} - 4.15\,m/s)$$

$$I = -3.12 \times 10^{-2} kg\,m/s$$

The rate of collisions is $10\,s^{-1}$. For a period of time Δt, there are

$$n_{collisions\,in\,\Delta t} = 10\,s^{-1}\Delta t$$

collisions. The total impulse during that period of time is:

$$I_{tot} = n_{collisions\,in\,\Delta t} I$$

The average applied force is:

$$F_{av} = \frac{I_{tot}}{\Delta t} = \frac{10\,s^{-1}\Delta t\,I}{\Delta t} = 10\,s^{-1}\,I$$

$$F_{av} = 0.312N$$

to the right.