PHY 180

Assignment 6 Solutions

DUE DATE: November 10, 2005

Question 1:

(a)

Since R = 20 cm then 2.7R = 54 cm. The percentage difference is $\frac{h-2.7R}{h} \times 100 = \frac{65-54}{65} \times 100 =$ 17%.

(b)

For the situation when the ball is just about to leave the track, after it leaves the loop it rises to a point where the bottom of the ball is at about the height of the top of the track or about 2 cm above the track. The centre of mass of the ball when it was going around the loop was about 2 cm below the track when the ball was at its highest. So the ball after leaving the loop rises to about a distance of 2R + 4 cm or 44 cm. So the energy loss is about $\frac{65-44}{65} \times 100 = 32\%$. Actually 32% represents an upper limit. If the ball falls over the end of the apparatus then it will still have some rotational and translational kinetic energy. The actual number (32%) is not required but only a number which is larger than the 17% of section (a). In fact you would expect a number substantially larger than 17% because when the ball is just about to leave the track in (a) only half of the distance has been covered and after the ball passes the highest point on the loop it continues to lose energy. (A simplistic explanation would conclude that the answer in (b) should be double the answer in (a) because of this approximate factor of 2 in distance.)

(c)

Since from our rough calculations the energy loss is 32% or a little less, it is obvious that our assumption that no energy is lost to the track is incorrect and this energy loss more than accounts for the “error” in (a).

Question 2: Chapter 8, #58

(a)

$$\begin{aligned}\mathbf{F} &= -\frac{dU}{dx} \\ &= -\frac{d}{dx}(-x^3+2x^2+3x)\hat{\mathbf{i}} \\ &= (3x^2-4x-3)\hat{\mathbf{i}}\end{aligned}$$

(b)

$$\begin{aligned}F &= 0 \\ (3x^2-4x-3) &= 0\end{aligned}$$

gives

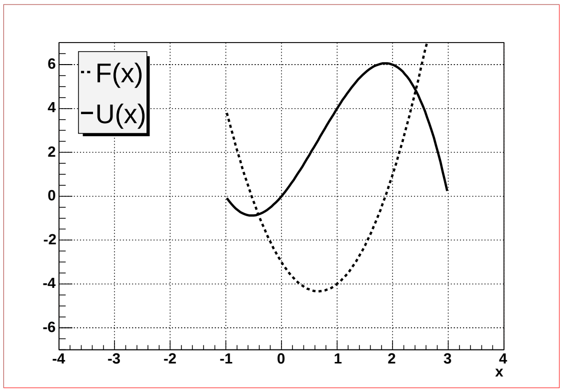
$$\begin{aligned}x &= \frac{4+\sqrt{52}}{6} \\ x &= 1.87\end{aligned}$$

and

$$\begin{aligned}x &= \frac{4-\sqrt{52}}{6} \\ x &= -0.535\end{aligned}$$

(c)

See plot below. Stable equilibrium occurs at $x = -0.535$ ($U(x)$ is at a minimum). Unstable equilibrium occurs at $x = 1.87$ ($U(x)$ is at a maximum).



Question 3: Chapter 10, #70

(a)
Use conservation of energy. Take zero point of potential energy to be where spring is at its unstretched equilibrium position.

Initially, there is energy stored in the stretched spring,

$$U_{s,i} = \frac{1}{2}kd^2$$

and potential energy due to gravity, i.e. the block is now at a height h above the zero potential energy level ($h = d\sin\theta$),

$$\begin{aligned} U_{g,i} &= mgh \\ &= mgd\sin\theta \end{aligned}$$

When the spring is again unstretched, the block is back at the equilibrium position and has only kinetic energy,

$$K_{block,f} = \frac{1}{2}mv^2$$

There is also kinetic energy due to the rotation of the pulley,

$$K_{rot,f} = \frac{1}{2}I\omega^2$$

Since there are no net external forces on the system we can write,

$$\begin{aligned} U_{s,i} + U_{g,i} &= K_{block,f} + K_{rot,f} \\ \frac{1}{2}kd^2 + mgd\sin\theta &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \end{aligned}$$

Using $v = R\omega$ and solving for ω,

$$\begin{aligned} \frac{1}{2}kd^2 + mgd\sin\theta &= \frac{1}{2}m(R\omega)^2 + \frac{1}{2}I\omega^2 \\ kd^2 + 2mgd\sin\theta &= \omega^2(mR^2 + I) \\ \omega^2 &= \frac{kd^2 + 2mgd\sin\theta}{mR^2 + I} \\ \omega &= \sqrt{\frac{kd^2 + 2mgd\sin\theta}{mR^2 + I}} \end{aligned}$$

(b)
Evaluating this with the values given,

$$
\begin{aligned}
\omega &= \sqrt{\frac{kd^2 + 2mgd\sin\theta}{mR^2 + I}} \\
\omega &= \sqrt{\frac{(50.0)(0.200)^2 + 2(0.500)(9.80)(0.200)\sin 37^\circ}{(0.500)(0.300)^2 + 1.00}} \\
\omega &= 1.74 \text{ rad/s}
\end{aligned}
$$

Question 4: Chapter 11, #14

We want to compute the magnitude of the angular momentum, i.e.

$$L = |\mathbf{r} \times \mathbf{p}|$$

and since **r** and **p** are perpendicular to each other,

$$\begin{aligned} L &= rp \\ &= rmv \end{aligned}$$

Now we need to find an expression for v.

The sum of forces in the y direction gives:

$$\begin{aligned} T\cos\theta &= mg \\ T &= \frac{mg}{\cos\theta} \end{aligned}$$

The sum of forces in the x direction gives:

$$\begin{aligned} T\sin\theta &= \frac{mv^2}{r} \\ T &= \frac{mv^2}{r\,\sin\theta} \end{aligned}$$

Eliminating T and solving for v gives,

$$\begin{aligned} \frac{mg}{\cos\theta} &= \frac{mv^2}{r\,\sin\theta} \\ v^2 &= \frac{gr\,\sin\theta}{\cos\theta} \\ v &= \sqrt{\frac{gr\,\sin\theta}{\cos\theta}} \end{aligned}$$

Now substituting into our equation for L and using $r = l\,\sin\theta$,

$$\begin{aligned} L &= rmv \\ &= rm\left(\sqrt{\frac{gr\,\sin\theta}{\cos\theta}}\right) \\ &= \sqrt{\frac{m^2gr^3\,\sin\theta}{\cos\theta}} \\ &= \sqrt{\frac{m^2g(l\,\sin\theta)^3\,\sin\theta}{\cos\theta}} \\ &= \sqrt{\frac{m^2gl^3\,\sin^4\theta}{\cos\theta}} \end{aligned}$$