PHY180 Assignment # 5 Solutions

Chapter 6, Number 42

distance to plate = 18.3 m
v_i = 40.2 m/s

from Example 6.13, the characteristics of the baseball are:

m = 0.145 kg
A = 4.2 x 10^{-3} m^2
D = 0.305 (drag coefficient)
R = $\frac{1}{2} D \rho A v^2$ (magnitude of resistive force)

from Problem 41 (b), the speed as a function of position is:

$v(x) = v_0 e^{-kx}$, if the resistive force is of the form $f = -kmv^2$

Question: What is the speed of the ball at the plate?

Solution:

To find k in the given equation for $v(x)$, equate the general resistive force to the resistive force from Example 6.13 (which is negative since the ball slows down due to air resistance):

$$-kmv^2 = -\frac{1}{2} D \rho A v^2$$

$$km = \frac{1}{2} D \rho A$$

$$k = \frac{D \rho A}{2m}$$

The equation for $v(x)$ becomes:

$$v(x) = v_0 e^{-kx}$$

$$v(x) = v_0 \exp\left[\frac{-D \rho A x}{2m}\right]$$

Substituting in the values given in the question, with $\rho = 1.20\, kg/m^3$ from the inside front cover of the textbook:

$$v(18.3\, m) = (40.2\, m/s) \exp\left[\frac{-(0.305)(1.20\, kg\, m^{-3})(4.2 \times 10^{-3}\, m^2)(18.3\, m)}{2(0.145\, kg)}\right]$$

$$v(18.3\, m) = 36.5\, m/s$$

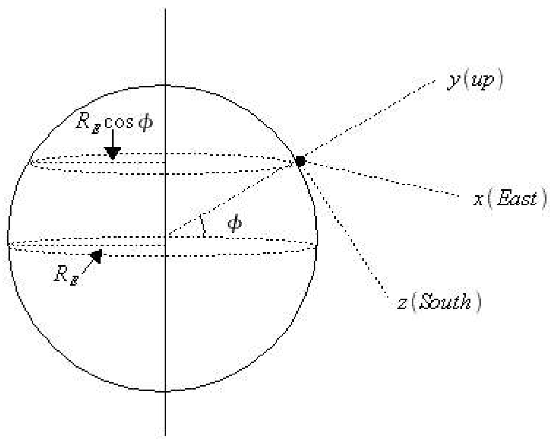
The speed of the baseball when it reaches home plate is 36.5 m/s.

Chapter 6, Number 66

The golfball is hit due south from a latitude of $\phi_i = 35.0^0 N$, with a range of 285 m. The initial direction of motion is $\theta_i = 48.0^0$ above the horizon.

(a) How long is the ball in flight?

Define the coordinate system such that x is eastward, y is upwards, and z is southward:



Once the ball leaves the Earth, it will have constant velocity in the x-direction (due to the rotation of the Earth at its initial latitude) and z-direction (due to the speed given to it by the golf club). This is then 3D projectile motion, with only the y-direction velocity changing due to the force of gravity. Use the range equation to find v_i:

$$R = v_i^2 \sin\frac{2\theta_i}{g}$$

$$v_i = \sqrt{\frac{gR}{\sin 2\theta_i}}$$

To find the time in the air, use constant acceleration equation: $y_f - y_i = v_{iy} t + \frac{1}{2} a_y t^2$

The ball starts and ends on the ground: $y_f = y_i = 0\,m$
Acceleration is due to gravity: $a_y = g$
And the initial y-velocity is the upwards component of the ball's total initial velocity: $v_{iy} = v_i \sin\theta_i$
Substituting all of this into the above equation gives:

$$0 = v_i(\sin\theta_i)t + \frac{1}{2} g t^2$$

$$0 = \sqrt{\frac{gR}{\sin 2\theta_i}}(\sin\theta_i)t + \frac{1}{2} g t^2$$

$$0 = \sqrt{\frac{(9.80\,m\,s^{-2})(285\,m)}{\sin(2(48.0^0))}}\sin(48.0^0)t + \frac{1}{2}(9.80\,m\,s^{-2})t^2$$

$$0 = (39.38\,m\,s^{-1})t + (4.90\,m\,s^{-2})t^2$$

$$0 = t((39.38\,m\,s^{-1}) + (4.90\,m\,s^{-2})t)$$

$$t = 0\, s\, or$$
$$t = \frac{39.38\, m\, s^{-1}}{4.90\, m\, s^{-2}}$$
$$t = 8.04\, s$$
The ball is in flight for 8.04 s.

(b) What is the eastward speed of the tee relative to the stars? Any point on the Earth undergoes uniform circular motion about the Earth's axis with a distance $R_E \cos\phi_i$ from the axis, and a period of one day. Use equation of circular motion:
$$v = \omega r$$
$$v = \frac{2\pi}{T} r$$
$$v = \frac{2\pi R_E \cos\phi_i}{T}$$
$$v = \frac{2\pi (6.37\times10^6\, m)\cos(35.0^0)}{(1\, day)(24\, h/day)(3600\, s/h)}$$
$$v = 379\, m/s$$
The eastward speed of the tee is 379 m/s.

(c) How much larger is the speed of the hole than the speed of the tee?

$$v_{hole} = \frac{2\pi R_E \cos\phi_{hole}}{T}$$
$$v_{tee} = \frac{2\pi R_E \cos\phi_{tee}}{T}$$
We need to know ϕ_{hole} . We know the hole is 285 m south of the tee. A circle around the Earth corresponds to 360^0, and $2\pi R_E$ (the perimeter of a circle of radius R_E), so we can find the difference in the latitude of the hole and the latitude of the tee:
$$\frac{\Delta\phi}{360^0} = \frac{285\, m}{2\pi R_E}$$
$$\Delta\phi = \left(\frac{(285\, m)}{2\pi(6.37\times10^6\, m)}\right)(360^0)$$
$$\Delta\phi = (2.563\times10^{-3})^0$$
The difference in velocity is:
$$\Delta v = \frac{2\pi R_E}{T}\left(\cos\phi_{hole} - \cos\phi_{tee}\right)$$
$$\Delta v = \frac{2\pi R_E}{T}\left(\cos(\phi_{tee} - \Delta\phi) - \cos\phi_{tee}\right)$$
$$\Delta v = \frac{2\pi R_E}{T}\left(\cos\phi_{tee}\cos\Delta\phi + \sin\phi_{tee}\sin\Delta\phi - \cos\phi_{tee}\right)$$

$\Delta\phi$ is very small, so $\cos\Delta\phi \approx 1$:

$$\Delta v = \frac{2\pi R_E}{T}\left(\cos\phi_{tee} + \sin\phi_{tee}\sin\Delta\phi - \cos\phi_{tee}\right)$$
$$\Delta v = \frac{2\pi R_E}{T}\left(\sin\phi_{tee}\sin\Delta\phi\right)$$
$$\Delta v = \frac{2\pi(6.37\times10^6 m)}{(1\,day)(24\,h/day)(3600\,s/h)}\sin(35.0^0)\sin(2.563\times10^{-3})$$
$$\Delta v = 1.19\times10^{-2} m/s$$

The hole moves 1.19 x 10^{-2} m/s eastward relative to the tee.

(d)How far to the west of the hole does the ball land?

The ball will have the same eastward velocity as the tee, so will lag the hole by Δv found in (c). Since this motion is at a constant speed:
$$\Delta x = \Delta v t$$
$$\Delta x = (1.19\times10^{-2} m s^{-1})(8.04 s)$$
$$\Delta x = 9.56\times10^{-2} m$$
The ball lands 9.56 cm to the west of the hole.

<u>Chapter 7, Number 42</u>

1 kcal = 4186 J
1 gram of fat releases 9.00 kcal
a student of mass 50.0 kg runs up 80 steps, each of height 0.150 m in 65.0 s.
energy efficiency in muscles is 20.0%

(a) How many times must she run up the stairs to lose 1 pound of fat?

Converting to SI:
1 lb = 0.453 kg
9.00 kcal = 3.767 x 10^4 J
To burn one pound of fat requires:
$$\frac{3.767\times10^4 J}{0.001\,kg}\times0.453\,kg = 1.706\times10^7 J$$
The energy that goes into climbing is 20.0% of what she burns:
$$E_{fat} = (0.200)(1.706\times10^7 J) = 3.412\times10^6 J$$
If the student climbs the stairs *N* times, the energy she needs to use is the energy needed to overcome gravity: $E_{stairs} = Nmgh$. To find *N*, we equate the energy from burning the fat to the energy needed to climb the steps:
$$E_{fat} = E_{stairs}$$
$$3.412\times10^6 J = Nmgh$$
$$N = \frac{(3.412\times10^6 J)}{mgh}$$
$$N = \frac{(3.412\times10^6 J)}{(50.0\,kg)(9.80\,m s^{-2})(80\,steps)(0.150\,m/step)}$$
$$N = 580$$

She must climb the steps 580 times to burn one pound of fat.

(b) What is her average power in Watts and horsepower?

The power output to climb one flight of stairs is:

$$P = \frac{mgh}{t}$$

$$P = \frac{(50.0\,kg)(9.80\,m\,s^{-2})(80\,steps)(0.150\,m/step)}{65.0\,s}$$

$$P = 90.5\,W$$

Converting to horsepower:

$$P = 90.5\,W \times \frac{1\,hp}{746\,W}$$

$$P = 0.121\,hp$$

The student's average power output is 90.5 W or 0.121 hp.

Chapter 7, Number 68

resistive force on the windmill is $R = \frac{1}{2} D \rho A v^2$

power available is $P = Rv = \frac{1}{2} D \rho \pi r^2 v^3$

Take: D = 1.00
r = 1.50 m
ρ = 1.20 kg/m^3

(a) If v = 8.00 m/s, what is the available power?

$$P = \frac{1}{2} D \rho \pi r^2 v^3$$

$$P = \frac{1}{2}(1.00)(1.20\,kg\,m^{-3})\pi(1.50\,m)^2(8.00\,m\,s^{-1})^3$$

$$P = 2.17 \times 10^3\,W$$

The available power is 2.17 x 10^3 W.

(b) If v = 24.0 m/s, what is the available power?

$$P = \frac{1}{2} D \rho \pi r^2 v^3$$

$$P = \frac{1}{2}(1.00)(1.20\,kg\,m^{-3})\pi(1.50\,m)^2(24.0\,m\,s^{-1})^3$$

$$P = 5.86 \times 10^4\,W$$

The available power is 5.86 x 10^4 W.