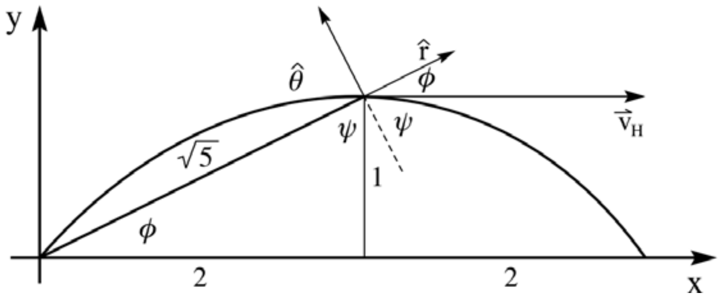
Assignment 4 2005 Solutions

Problem 1



(a) $\mathbf{R} = \dfrac{\mathbf{v^2 \sin(2\theta)}}{\mathbf{g}}$ and

$$\mathbf{H} = \frac{\mathbf{v^2 \sin^2\theta}}{2\mathbf{g}}$$

$$\frac{\mathbf{v^2 \sin^2\theta}}{2\mathbf{g}} = \frac{1}{4}\frac{\mathbf{v^2 \sin(2\theta)}}{\mathbf{g}}$$

$$\frac{\mathbf{\sin^2\theta}}{2} = \frac{1}{4}\,\mathbf{2\sin\theta\cos\theta}$$

$\mathbf{\sin\theta} = \mathbf{\cos\theta}$ or $\mathbf{\tan\theta} = 1$. Therefore $\boldsymbol{\theta}$ = 45°.

(b)

The plane is hit for $\mathbf{y} = \mathbf{H}$ and at that point $\mathbf{v_{Hy}} = 0$ and $\mathbf{v_{Hx}} = \mathbf{v\cos(}45°\mathbf{)}$

$$\vec{\mathbf{v}}_{\mathbf{H}} = \mathbf{v\cos(}45°\mathbf{)}\,\hat{\mathbf{i}}$$

In terms of $\hat{\mathbf{r}}$ and $\widehat{\boldsymbol{\theta}}$, from the diagram

$\hat{\mathbf{r}}$ makes an angle $\boldsymbol{\varphi}$ above the horizontal

$\widehat{\boldsymbol{\theta}}$ makes an angle 90 + $\boldsymbol{\varphi}$ above the horizontal or $\boldsymbol{\psi}$ below the horizontal. Therefore

$$\vec{\mathbf{v}}_{\mathbf{H}} = \mathbf{v\cos(45°)\cos(\varphi)\,\widehat{r} - v\cos(45°)\cos(\psi)\,\widehat{\theta}}$$

$$\vec{\mathbf{v}}_{\mathbf{H}} = \mathbf{v}\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\frac{\mathbf{2}}{\sqrt{\mathbf{5}}}\widehat{\mathbf{r}} - \mathbf{v}\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\frac{\mathbf{1}}{\sqrt{\mathbf{5}}}\widehat{\boldsymbol{\theta}}$$

$$\vec{\mathbf{v}}_{\mathbf{H}} = 0.63\,\mathbf{v}\,\widehat{\mathbf{r}} - 0.316\,\mathbf{v}\,\widehat{\boldsymbol{\theta}}$$

Problem 2

(a) The component of **a** parallel to the slope is $\mathbf{g\sin\theta}$.

$\mathbf{a} = \mathbf{g\sin\theta}$

$\mathbf{v_i} = 0$

$\mathbf{x} = \mathbf{L}$

$\mathbf{t} = \mathbf{?}$

$\mathbf{x} = \mathbf{v_i\, t} + \tfrac{1}{2}\mathbf{a\, t^2}$

$\mathbf{L} = \mathbf{0} + \tfrac{1}{2}\mathbf{g\sin\theta}$

$$\mathbf{t} = \sqrt{\frac{\mathbf{2L}}{\mathbf{g\sin\theta}}}$$

(b) For the variable **d** and **h** constant

You get the minimum time for $\mathbf{t} = \sqrt{\frac{\mathbf{2L}}{\mathbf{g\sin\theta}}}$ when $\mathbf{\sin\theta = 1}$ or when $\boldsymbol{\theta} = 90°$.

$\mathbf{t} = \sqrt{\frac{\mathbf{2L}}{\mathbf{g}}}$ and now $\mathbf{L = h}$ and so

$$\mathbf{t} = \sqrt{\frac{\mathbf{2h}}{\mathbf{g}}}$$

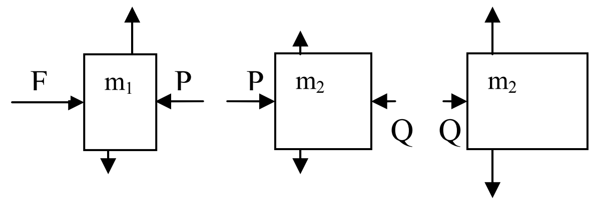
(c) For variable **h** and **d** constant $\frac{\mathbf{d}}{\mathbf{L}} = \mathbf{\cos\theta}$ and therefore $\mathbf{L} = \frac{\mathbf{d}}{\mathbf{\cos\theta}}$.

$$\mathbf{t} = \sqrt{\frac{2\mathbf{L}}{\mathbf{g\sin\theta}}} = \sqrt{\frac{2\mathbf{d}}{\mathbf{g\sin\theta\cos\theta}}} = \sqrt{\frac{2\mathbf{d}}{\mathbf{g}\,\tfrac{1}{2}\mathbf{\sin2\theta}}} = 2\sqrt{\frac{\mathbf{d}}{\mathbf{g\sin2\theta}}}$$

This is a minimum when $\mathbf{\sin2\theta} = 1$ or when $\theta = 45°$ and therefore

$$\mathbf{t} = 2\sqrt{\frac{\mathbf{d}}{\mathbf{g}}}$$

Ch. 5 # 54



First, set up the free-body diagrams for the blocks m_1, m_2 and m_3:

a) Block 1: 18 - P = (2 kg) a
   Block 2: P - Q = (3 kg) a
   Block 3: Q = (4 kg) a

   So we just have to solve 3 equations for 3 unknowns. The easiest way is to add all the equations, giving:

   18 = (9 kg) a
   so

   $\boxed{a = 2.00 \text{ m/s}^2}$

(Note that to do this problem we could also have treated all three blocks as a single system of mass $m = m_1+m_2+m_3$, subjected to a net force of F = 18 N to find acceleration)

b) To find the resultant forces, just use $F_{net} = m\ a$:

   Block 3: $F_{net} = m_3\ a = (4 \text{ kg})(2 \text{ m/s}^2) = \boxed{8.00 \text{ N}}$

   Block 2: $F_{net} = m_2\ a = (3 \text{ kg})(2 \text{ m/s}^2) = \boxed{6.00 \text{ N}}$

   Block 1: $F_{net} = m_1\ a = (2 \text{ kg})(2 \text{ m/s}^2) = \boxed{4.00 \text{ N}}$

c) To find the contact forces, we can use the equations from part (a):
   Contact force between blocks 2 and 3 = Q

   $Q = (4 \text{ kg})\ a = (4\text{kg})(2 \text{ m/s}^2) = \boxed{8.00 \text{ N}}$

   Contact force between blocks 1 and 2 = P
   $P = Q + (3 \text{ kg})\ a = 8 + (3 \text{ kg})(2 \text{ m/s}^2) = \boxed{14.0 \text{ N}}$

d) The 3-kg block models the heavy block of wood. The contact force on your back is represented by Q, which is much less than the force F. The difference between F and Q is the net force causing acceleration of the 5-kg pair of objects. The acceleration is real and nonzero, but lasts for such a short time that it is never associated with a large velocity. The frame of the building and your legs exert forces, small relative to the hammer blow, to bring the partition, block and you to rest again over a time large relative to the hammer blow.

Ch. 10 # 46

Choose the height at which the two masses pass as the position of zero gravitational potential energy. Then use conservation of energy to find the speeds of the blocks as they pass. (Note that since they are attached by the rope, the speeds of the two blocks will be equal at all times).

The blocks start from rest, so the initial kinetic energies are zero for both blocks. Also, the pulley is not rotating initially, so the rotational kinetic energy of the pulley is zero. The blocks are initially 3 metres apart, so when they pass, the lower block will have moved up 1.5 metres, and the higher block will have moved down 1.5 metres.
Thus initially we have:

$KE_i = 0$;
$PE_i = m_1(1.5\text{ m})g + m_2(-1.5\text{ m})g$;

The final gravitational potential energy of the blocks is zero, so the total final energy is

$$KE_{tot} = KE_1 + KE_2 + KE_{rot}$$
$$= \tfrac{1}{2} m_1 v^2 + \tfrac{1}{2} m_2 v^2 + \tfrac{1}{2} I\, T^2$$

For a uniform disk, $I = \tfrac{1}{2}MR^2$;
And recall that the speed of the edge of the disk (which is the same as the speed of the masses) can be written as $v = R\,T$, so we can substitute $T = v/R$ into the above expression for KE_{rot}
Thus from conservation of energy,

$$1.5\, m_1 g + (-1.5)\, m_2 g = \tfrac{1}{2} m_1 v^2 + \tfrac{1}{2} m_2 v^2 + \tfrac{1}{2} I\, T^2$$
$$= \tfrac{1}{2} m_1 v^2 + \tfrac{1}{2} m_2 v^2 + \tfrac{1}{2}\left(\left(\tfrac{1}{2}\right) MR^2\right)(v/R)^2$$
$$= v^2 \left(\tfrac{1}{2} m_1 + \tfrac{1}{2} m_2 + \tfrac{1}{4} M\right)$$

Substituting m1 = 15 kg, m2 = 10 kg and M = 3 kg,

$$1.5\, g\, (15 - 10) = v^2 \left(\tfrac{1}{2} (15 + 10) + \tfrac{1}{4} (3)\right)$$
$$(1.5)(9.8)(5) = v^2 \left(\tfrac{1}{2} (25) + \tfrac{3}{4} \right)$$

$\boxed{v = 2.36\text{ m/s}}$