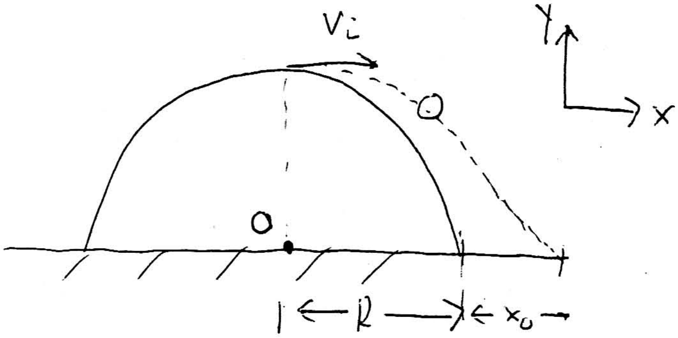
PHY 180F Assignment #3 Answers

1. Ch. 4, #62.



The y-direction motion of the ball is

$$y_b = y_0 + \cancel{v_{0y}}^{0} t + \frac{1}{2} a_y t^2$$
$$= R + 0 - \frac{1}{2} g t^2 \quad (1)$$

The x-direction motion of the ball is

$$x_b = x = v_{0x} t = v_i t \quad (2)$$

From (2), $t = \frac{x}{v_i}$ → sub this into (1)

$$y_b = R - \frac{1}{2} \frac{g x^2}{v_i^2} \quad (3)$$

The surface of the rock lies on

$$y_r^2 + x^2 = R^2$$

At $x = 0$, $y_r = y_b$, but for ALL $x > 0$, we require $y_b > y_r$

$$\therefore \quad R - \frac{1}{2} \frac{g x^2}{v_i^2} > \sqrt{R^2 - x^2} \qquad (\text{note } y_r, y_b \geq 0 \text{ for } x \geq 0)$$

$$\left(R - \frac{1}{2} \frac{g x^2}{v_i^2}\right)^2 > R^2 - x^2$$

$$\cancel{R^2} - \frac{g x^2 R}{v_i^2} + \frac{g^2 x^4}{4 v_i^4} > \cancel{R^2} - x^2$$

Cancelling factors of x^2 and multiplying through by $4v_i^4$ (all these terms are positive so the inequality does not change), and rearranging

$$g^2 x^2 + 4 v_i^4 > 4 g v_i^2 R$$
$$g^2 x^2 + 4 v_i^2 (v_i^2 - gR) > 0 \quad (4)$$

The inequality in (4) will be satisfied for all $x \geq 0$ provided

$v_i^2 - gR \geq 0$

$\therefore$ the minimum v_i for this to be true is

$v_i^2 \geq gR$

$v_i \geq \sqrt{gR}$

b) Sub the answer from part a) into (3) and set $y_b = 0$

$0 = R - \frac{1}{2}\frac{\cancel{g}x^2}{\cancel{g}R}$

$x^2 = 2R^2$

$x = \sqrt{2}R$

The distance x_0 from the base of the rock is

$x_0 = \sqrt{2}R - R = R(\sqrt{2} - 1)$.

2. Ch 4, # 64 (very similar to question #1)

$y_{bank}^2 = 16x \quad \rightarrow y_{bank} = -\sqrt{16x} = -4\sqrt{x} \quad (1)$

(take the origin to be at the edge of the road, where it meets the bank)

for the melon,

(2) $y = -\frac{1}{2}gt^2$

(3) $x = v_i t \quad \rightarrow t = \frac{x}{v_i} \rightarrow$ sub into (2)

(4) $y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)$

Set (4) = (1) to find where the melon strikes the bank

$-4\sqrt{x} = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)$

$16x = \frac{1}{4}g^2\frac{x^4}{v_i^4} \quad \rightarrow x\left(\frac{g^2x^3}{v_i^4} - 64\right) = 0$

So, $x^3 = \frac{64v_i^4}{g^2}$ or $x = 0$ (trivial)

$\therefore \; x^3 = \dfrac{64(10)^4}{(9.80)^2} \quad \rightarrow \quad x = 18.8 \text{ m}$

Use this answer in (4)

$$y = -\frac{1}{2} g \left(\frac{(18.8)^2}{(10.0)^2} \right) = -17.3 \text{ m}.$$

$\therefore$ the melon hits the bank at $(18.8, -17.3)$ m.

3. Ch. 5, #30.

a) From p. 129 of Serway, the acceleration of both masses is

$$a = \frac{m_2 - m_1}{m_1 + m_2} g \quad , \text{with } m_2 > m_1$$

m_2 accelerates in the negative y-direction, m_1 in the positive y-d

At $y = 0$, m_1 is given an initial velocity of $-2.4 \frac{m}{s}$. The constant acceleration of m_1 is $\frac{m_2 - m_1}{m_1 + m_2} g = \frac{7\,kg - 2\,kg}{7\,kg + 2\,kg} 9.8 \frac{m}{s^2} = 5.44 \frac{m}{s^2}$

$$v_f^2 = v_i^2 + 2a_y (y_f - y_i)$$

with $a_y = +5.44 \frac{m}{s}$, $y_i = 0$, $v_i = -2.4 \frac{m}{s}$, we want to find y_f when $v_f = 0 \frac{m}{s}$ $\rightarrow 0 = v_i^2 + 2a_y y_f$

$\cancel{0 = (2.4 \frac{m}{s})^2 + 2\, 5.44 \frac{m}{s} (y_f - 0)}$ $\quad y_f = \dfrac{-v_i^2}{2a_y} = \dfrac{-(-2.4 \frac{m}{s})^2}{2 \cdot 5.44 \frac{m}{s^2}}$

$= -0.529$ m

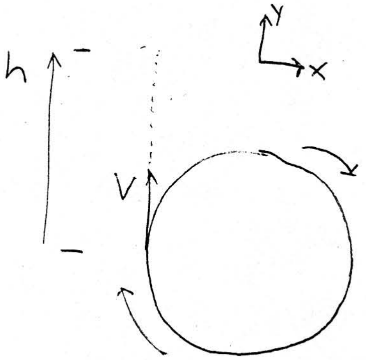
$= -0.53$ m

$\therefore$ m_1 falls 0.53 m below its initial level

b) $v_f = v_i + a_y t$, with $t = 1.80$ s, $v_i = -2.4 \frac{m}{s}$, $a_y = +5.44 \frac{m}{s^2}$

$v_f = +7.40 \frac{m}{s}$ (up)

Ch 10, # 64.



The drop leaves the wheel with velocity $v_1 \hat{Y}$. Take $y=0$ to be the point where $U=0$

Then $E_f = E_i$, with $i \rightarrow$ where drop leaves the wheel, $f \rightarrow$ max height of drop

$$K_f + U_f = K_i + U_i$$

$$mgh_1 = \frac{1}{2} m v_1^2$$

then $v_1 = \sqrt{2gh_1}$

and the angular velocity of the wheel is $\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}$

Similarly, for the second drop, $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}$ with $h_2 < h_1$ (so $\omega_2 < \omega_1$)

$$2\alpha\Delta\theta = \omega_2^2 - \omega_1^2$$

The two drops leave the wheel on consecutive turns, $\therefore \Delta\theta = 2\pi$

$$\alpha = \frac{1}{2 \cdot 2\pi}\left(\frac{2gh_2}{R^2} - \frac{2gh_1}{R^2}\right) = \frac{g(h_2 - h_1)}{2\pi R^2}$$

(α is negative, since $h_2 < h_1$).