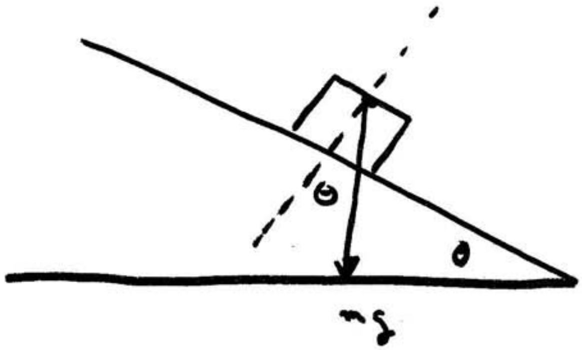
1

(a) $\parallel$ TO THE SLOPE



$v_i = 0$

$v_f = 30.0 \ \mathrm{ms^{-1}}$

$t = 6.00 \ \mathrm{s}$

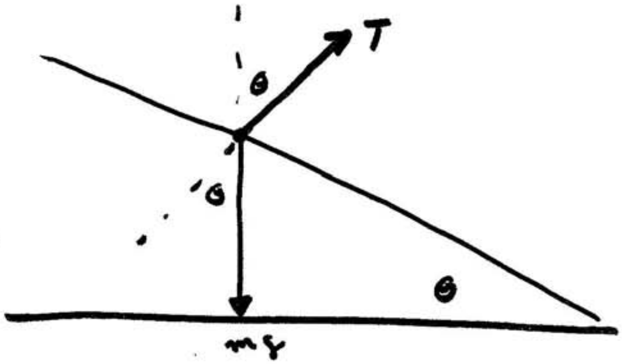
$a = ?$

$v_f = v_i + at$

$30.0 = 0 + a \times 6.00$

$a = 5.00 \ \mathrm{ms^{-1}}$

<u>FOR THE TOY</u>



THE ONLY FORCE $\parallel$ SLOPE IS $mg \sin\theta$

BUT $F = ma$

$\cancel{m} g \sin\theta = \cancel{m} a$

$\sin\theta = \dfrac{6.00}{9.80} \quad \therefore \ \theta = 30.7^\circ$

(b) $\perp$ TO THE SLOPE $\sum F = 0$

$\therefore \quad T = mg\cos\theta$

$= 0.106 \times 9.80 \times \cos(30.7)$

$T = 0.843 \ \mathrm{N}$

(2)

a) THE OBJECT IS ALWAYS ROTATING ABOUT THE POINT IN CONTACT WITH THE FLOOR BUT WE MAY BREAK UP THE MOTION INTO TWO PARTS; ROTATION ABOUT THE CENTRE OF MASS AND TRANSLATION OF THE CENTRE OF MASS. THE ANGULAR VELOCITY ABOUT THE CENTRE OF MASS IS EQUAL TO THE ANGULAR VELOCITY ABOUT P.

$I_{cm} = \frac{1}{2} MR^2$ FOR A DISK

$v_{cm} = R\omega$

$$KE\,(TOTAL) = KE\,(ROT.) + KE\,(TRANS)$$

$$= \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} M v_{cm}^2$$

$$= \frac{1}{2}\left(\frac{1}{2} MR^2 \left(\frac{v_{cm}}{R}\right)^2\right) + \frac{1}{2} M v_{cm}^2$$

$$= \frac{1}{4} M v_{cm}^2 + \frac{1}{2} M v_{cm}^2$$

$$\boxed{KE\,(TOTAL) = \frac{3}{4} M v_{cm}^2}$$

b) FOR ROTATION ABOUT THE POINT P.

FROM THE PARALLEL AXIS THEOREM

$I_p = I_{cm} + MD^2$ where $D = R$

$$I_p = \frac{1}{2} MR^2 + MR^2 = \frac{3}{2} MR^2$$

$$KE\,(TOTAL) = KE\,(ROT) = \frac{1}{2} I_p \omega^2$$

$$= \frac{1}{2}\left(\frac{3}{2}\right) MR^2 \left(\frac{v_{cm}}{R}\right)^2$$

$$KE(TOT) = \frac{3}{4} M v_{cm}^2$$

c) YES THEY SHOULD BE THE SAME SINCE THEY BOTH DESCRIBE THE MOTION OF THE SAME PHYSICAL OBJECT. BREAKING UP THE MOTION INTO TRANSLATION OF THE CENTRE OF MASS PLUS ROTATION ABOUT THE CENTRE OF MASS IS A CALCULATIONAL CONVENIENCE.

d) $$y_P = R + R\sin\left(\frac{v_{cm}}{R}t - \frac{\pi}{2}\right)$$

or $$y_P = R + R\sin\left(\frac{v_{cm}}{R}t + \frac{3\pi}{2}\right)$$

or $$y = R + R\cos\left(\frac{v_{cm}}{R}t \pm \pi\right)$$

or $$y = R - R\cos\left(\frac{v_{cm}}{R}t\right)$$

(a) 3

THE ARTILLERY SHELLS MUST CLEAR THE TOP OF THE BUILDING WITH COORDINATES $(350, h)$ IF THE ORIGIN IS AT THE MOUTH OF THE CANNON.

SINCE THE SOLDIERS HIT THE BUNKER EACH TIME, THE ANGLE FROM THE HORIZONTAL AT WHICH THE ARTILLERY SHELLS ARE FIRE CAN BE FOUND FROM:

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$2\theta = \sin^{-1}\left(\frac{gR}{v_i^2}\right) = \sin^{-1} \frac{9.8 \times 500}{72^2}$$

$$2\theta = 70.946$$

$$\theta = 35.47 \quad \text{DEGREES}$$

<u>NOTE</u>: WE USE THIS ANGLE, NOT ITS COMPLEMENT BECAUSE WE WANT TO MAKE SURE THAT THE HOTEL IS NEVER HIT.

THE EQUATION FOR THE PATH OF THE PROJECTILE IS:

$$y = (\tan\theta)\, x - \left(\frac{g}{2 v_i^2 \cos^2\theta}\right) x^2$$

FOR $x = 350$, $y = \tan(35.47) \times 350 - \dfrac{9.8 \times (350)^2}{2 \times 72^2 \cos^2(35.47)}$

$$= 249.38 - 174.57$$

$$= 74.81$$

$\therefore$ THE MAXIMUM HEIGHT OF THE HOTEL IS <u>75 m</u>

(b) WHEN THE ARTILLERY SHELLS ARE FIRED

$$v_x = 72 \cos(35.47) = 58.6 \ ms^{-1}$$

$$v_y = 72 \cos(35.47) = 41.8 \ ms^{-1}$$

WHEN THEY HIT THE GROUND

$$v_x = 58.6 \ ms^{-1}$$
$$v_y = -41.8 \ ms^{-1}$$

SINCE THE ARTILLERY SHELLS HIT ON THE x-AXIS, AT THAT SPOT

$$\hat{r} = \hat{i}$$
$$\hat{\theta} = j$$

$$\therefore \vec{v} = 59\hat{r} - 42\hat{\theta}$$