UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2004

First Year - Engineering Science

PHY180F - PHYSICS I - MECHANICS

Exam Type: C

Examiner: J. M. Pitre

Duration 2½ hours

Do all questions. All questions are of equal value. There are 80 possible marks.

Start the answer to each question on a new page.

You do not need to do the problems in order.

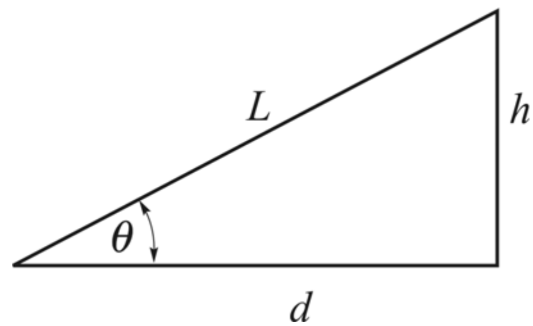
Use g = 9.80 ms^{-2} for the acceleration due to gravity in all problems.

Note that: $\sin 2\theta = 2\sin\theta\cos\theta$

Note that the roots a quadratic equation are: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1) The combination of an applied force and a frictional force produces a constant torque of 36.0 N·m on a wheel rotating about a fixed axis. The applied force acts for 6.00 s. During this time the angular speed of the wheel increases from 0.00 to 10.0 rad/s. The applied force is then removed and the wheel comes to rest in 60.0 s.
   a) What is the moment of inertia of the wheel?
   b) What is the magnitude of the frictional torque?
   c) What is the total number of revolutions that the wheel makes from the start of the problem to the end of the problem?

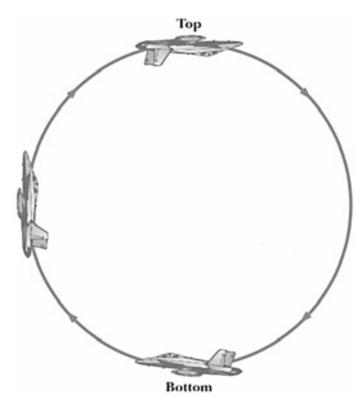
2) An object is sliding down a frictionless plane. The length of the plane is given by ***L***, the vertical and horizontal projections are given by ***h*** and ***d*** respectively and the angle of inclination is ***θ***.



   a) If an object starts from rest, find the time ***t*** for the object to slide down the entire plane in terms of the given quantities.
   b) If the angle ***θ*** is varied and only the vertical projection ***h*** remains constant, find the minimum time for the object to travel the length of the plane in terms of quantities which do not vary.
   c) If the angle ***θ*** is varied and only the horizontal projection ***d*** remains constant, find the minimum time for the object to travel the length of the plane in terms of quantities which do not vary.

3) A cannon is firing artillery shells at an enemy. The angle of elevation **θ** of the barrel of the cannon and the initial speed **v** of the artillery shells cannot be varied. Enemy soldiers on the ground can just reach a distance **X** from the cannon without being hit. Enemy aircraft are told that they will not be hit if they fly at an altitude of at least $\frac{1}{4}\mathbf{X}$. Take the origin to be at the cannon.
   a) Showing your work, derive the angle **θ**.
   b) An enemy aircraft flies just low enough to be hit by an artillery shell. Determine the velocity of the artillery shell in terms of the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ (cylindrical coordinates), where $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are defined by the position vector $\vec{\mathbf{r}}$ from the origin to the point where the aircraft is hit.

4) A pilot of mass **m** in a jet aircraft executes a loop-the-loop, as shown in the diagram. In this maneuver, the aircraft moves in a vertical circle of radius 2.70 km at a constant speed of 225 $m \cdot s^{-1}$. In this problem you should explain the steps in your solution and you should express your answers in terms of the weight of the pilot **mg**. What is the force exerted by the seat on the pilot:

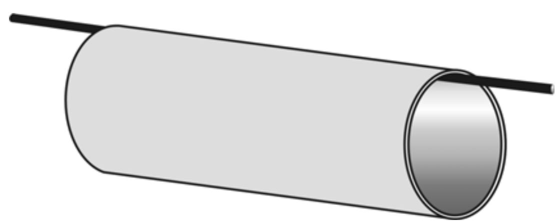


   a) at the bottom of the loop?
   b) at the top of the loop?

5) A rocket is to be used in outer space where gravity can be ignored. For this problem you can assume that all the fuel is consumed when the rocket accelerates. When type **A** fuel is used, half the rocket's initial mass must be fuel in order for the rocket to accelerate from rest to a speed **X** $m{\cdot}s^{-1}$ (i.e. the ratio of the mass of the fuel to that of the empty rocket is 1). When type **B** fuel is used, the speed of the ejected gas is three times the speed of ejected gas for type **A** fuel. If type **B** fuel is used to accelerate the rocket from rest to a speed of **X** $m{\cdot}s^{-1}$, what is the ratio of the mass of type **B** fuel to that of the empty rocket?

6) An elevator car has a mass of 1.60×10^3 kg and is carrying passengers having a combined mass of 200 kg. A motor pulls the car up by a cable and a constant frictional force of 4.00×10^3 N retards its upward motion.
   a) What power must the motor deliver to lift the elevator car and passengers at a constant speed of 3.00 $m{\cdot}s^{-1}$?
   b) If the motor is accelerating the elevator car and passengers with an upward acceleration of 1.00 $m{\cdot}s^{-2}$, what power must the motor deliver at the instant the speed of the elevator is 3.00 $m{\cdot}s^{-1}$?

7) The mass of the earth is $\mathbf{M_E}$ and the distance from the centre of the earth to the surface is $\mathbf{R_E}$. An object of mass **m** is shot straight up from the surface of the earth. You may neglect air resistance and the rotation of the earth. The escape velocity is the minimum speed at which the object must be shot so that it can escape to an infinite distance from the earth.
   a) Find the escape velocity for this object explaining all the steps in your solution.
   b) If the initial speed of the object is one half of the escape velocity then the object will rise to some maximum height **h** where **h** is measured from the centre of the earth to the highest position of the rocket. Express **h** as a multiple of $\mathbf{R_E}$.



8) A hollow cylinder of mass **M**, radius **R** and walls of negligible thickness (like an empty tin can with no ends) is suspended over a thin rod and you observe that it oscillates with a period **T**. You are now given a solid disc of mass **M** and radius **R**. You are asked to drill a small hole in the disc and suspend the disc from the thin rod. How far from the centre of the disc should you drill the hole so that the period of oscillation will be the same as the period **T** observed for the hollow cylinder? Assume that you can ignore the size of the hole and please explain the steps in your derivation.