Solutions to assignment 8

PST

November 18, 2004

1

First we calculate how fast the basketball comes off the floor. We know it rises about 0.5 metres, so by conservation of energy

$$\begin{aligned} \frac{1}{2} m_2 v_{2i}^2 &= m_2 g h \\ v_{2i} &= \pm\sqrt{2 \times 9.8 \times 0.5} \\ |v_{2i}| &\approx 3.13\text{m/s}\,. \end{aligned}$$

Now we treat the motion of the two balls carefully. Assume that, unlike the collision with the floor, the collision of the two balls is elastic. From Eq. 9.21 in the textbook

$$v_{2f} = \frac{2m_1}{m_1 + m_2}\, v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}\, v_{2i}\,.$$

The tennisball falls one metre, so by conservation of energy $v_{1i} = \sqrt{2 \times 9.8 \times 1.0} \approx 4.43\text{m/s}$, where we've assigned the direction 'down' positive velocity. In this coordinate system then, $v_{2i} \approx -3.13\text{m/s}$. The masses were measured to be $m_1 = 57\text{g}$ and $m_2 = 625\text{g}$. Putting these values into Eq. 9.21 gives

$$v_{2f} \approx \frac{2 \times 57}{57 + 625}\, 4.43 + \frac{625 - 57}{57 + 625}\, (-3.13)\,,$$

and so $v_{2f} \approx -1.86\text{m/s}$. The first term contributes quite a lot to this value, so the mass of the tennisball is not negligible. Using conservation of energy one more time, we can find the height the basketball should reach

$$\begin{aligned} \frac{1}{2} m_2 v_{2f}^2 &= m_2 g h \\ h &= \frac{(-1.86)^2}{2 \times 9.8} \\ h &\approx 0.18\text{m}\,. \end{aligned}$$

So the basketball rises roughly 18cm, which is a little less than half of 0.5m. Both students' observations were correct.

2

a)

Using the equations for elastic collisions in chapter 9 of the textbook

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\, v_{1i} + \frac{2m_2}{m_1 + m_2}\, v_{2i}\,.$$

The speed of the astroblaster at the ground, having fallen one metre, can be found using conservation of energy or from the equations of constantly accelerated motion. Either way, one finds

$$v^2 = 2gh = 2g.$$

- **object 1 is the 63g ball and object 2 is the earth**

  The relevant quantities are $m_1 = 63$, $v_{1i} = v$, $m_2 = \infty$, and $v_{2i} = 0$. Putting these into the above equation gives

  $$\begin{aligned} v_{1f} &= \frac{63-\infty}{63+\infty}\, v + \frac{2\infty}{63+\infty}\, 0 \\ &= -v \end{aligned}$$

- **object 1 is the 27g ball and object 2 is the 63g ball**

  The relevant quantities are $m_1 = 27$, $v_{1i} = v$, $m_2 = 63$, and $v_{2i} = -v$. Putting these into the above equation gives

  $$\begin{aligned} v_{1f} &= \frac{27-63}{27+63}\, v + \frac{2\times 63}{27+63}\,(-v) \\ &= -\frac{162}{90} v \end{aligned}$$

- **object 1 is the 9.4g ball and object 2 is the 27g ball**

  The relevant quantities are $m_1 = 9.4$, $v_{1i} = v$, $m_2 = 27$, and $v_{2i} = -162v/90$. Putting these into the above equation gives

  $$\begin{aligned} v_{1f} &= \frac{9.4-27}{9.4+27}\, v + \frac{2\times 27}{9.4+27}\,\big(-\frac{162}{90}v\big) \\ &= -3.15v \end{aligned}$$

Now use conservation of energy for the 9.4g ball.

$$\begin{aligned} \frac{1}{2}m(-3.15v)^2 &= mgh \\ h &= \frac{9.92v^2}{2g}, \end{aligned}$$

but from above the initial speed is such that $v^2 = 2g$, so

$$h = \frac{9.92\times 2g}{2g} = 9.9\text{m}$$

b)

Now assume that for each successive stage, m_1 is negligible compared to m_2. This means we can set $m_2 = \infty$ in the collision equation

$$\begin{aligned} v_{1f} &= \frac{m_1 - \infty}{m_1 + \infty} v_{1i} + \frac{2\infty}{m_1 + \infty} v_{2i} \\ &= -v_{1i} + 2v_{2i} \end{aligned}$$

Following through the same steps as in (a), we can now ignore the masses (since m_1 is always negligible) and just use this equation to find the speeds

- **object 1 is the 63g ball and object 2 is the earth**

  $v_{1i} = v$ and $v_{2i} = 0$ so

  $$v_{1f} = -v + 2 \times 0 = -v \tag{1}$$

- **object 1 is the 27g ball and object 2 is the 63g ball**

  $v_{1i} = v$ and $v_{2i} = -v$ so

  $$v_{1f} = -v + 2 \times (-v) = -3v \tag{2}$$

- **object 1 is the 9.4g ball and object 2 is the 27g ball**

  $v_{1i} = v$ and $v_{2i} = -3v$ so

  $$v_{1f} = -v + 2 \times (-3v) = -7v \tag{3}$$

Therefore, the little ball has speed $-7v$ and the height it reaches is

$$h = \frac{(-7v)^2}{2g} = \frac{49v^2}{2g} = \frac{49(2g)}{2g} = 49, \tag{4}$$

since from above $v^2 = 2g$. Thus the little ball skyrockets to 49m.

3

When an ice cube leaves the track it has a velocity given by conservation of energy

$$\begin{aligned} \frac{1}{2}mv^2 &= mgh \\ v &= \sqrt{2gh} \end{aligned}$$

An ice cube's horizontal speed is $v \cos\theta$, which is constant during it's flight. Since it rebounds off the wall with half it's initial (horizontal) speed, the change in momentum is

$$\Delta p = mv\cos\theta - (-\frac{1}{2}mv\cos\theta) = \frac{3}{2}mv\cos\theta.$$

The time between collisions is equal to the period $\Delta t = 1/f$, where f is the frequency of collisions. The average force is

$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{\frac{3}{2}mv\cos\theta}{\frac{1}{f}} = \frac{3fm\sqrt{2gh}\cos\theta}{2}.$$

The initial data are $m = 0.00500\text{kg}$, $h = 1.50\text{m}$, $f = 10.0\text{Hz}$, $\theta = 40.0°$, which gives

$$\overline{F} = 0.312\text{N}.$$

4

a)

The initial horizontal speed of an element Δm of falling sand is 0. The final horizontal speed is $v = 0.750\text{m/s}$. Thus

$$\Delta p_x = \Delta m \times v - \Delta m \times 0 = v\Delta m,$$

so the rate of change of momentum in the horizontal direction is

$$\frac{\Delta p_x}{\Delta t} = v\frac{\Delta m}{\Delta t}.$$

In the limit $\Delta t \to 0$, this becomes

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = v\frac{\mathrm{d}m}{\mathrm{d}t} = 0.750 \times 5.00 = 3.75\text{kg}\,\text{m/s}^2.$$

b)

Once an element of sand hits the conveyor, only friction accelerates it horizontally. Since Newton's second law is in general $\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$, the force of friction f must be equal to rate of change of momentum. Thus $f = 3.75\text{N}$.

c)

The only forces acting upon the belt are the external force and the frictional reaction force. By Newton's third law, the reaction force is $-f$ (taking right as postive, as in the diagram in the text). Since the conveyor is moving at constant speed, it is not accelerating, and by Newton's second law

$$F_{\text{ext}} - f = 0.$$

Thus $F_{\text{ext}} = 3.75\text{N}$.

d) The work done by the external force in one second is equal to the power exerted by that force. For a constant force, work is $W = F \times \Delta x$, and power is work over time, so

$$P_{\text{ext}} = F_{\text{ext}} \times \frac{\Delta x}{\Delta t} = F_{\text{ext}} \times v = 3.75 \times 0.75 = 2.81\text{Watts}.$$

So the work done in one second is 2.81J. Note that comparing parts (a), (b) and (c), we have $F_{\text{ext}} = v\frac{\mathrm{d}m}{\mathrm{d}t}$, so that

$$P_{\text{ext}} = v^2 \frac{\mathrm{d}m}{\mathrm{d}t}.$$

e)

The kinetic energy aquired by an element Δm of sand is entirely due to the horizontal motion. Since it starts with zero horizontal speed, the energy aquired is just the final kinetic energy $\frac{1}{2}\Delta m v^2$. The rate at which it aquires energy is just this quantity divided by Δt. As above, we can take the limit $\Delta t \to 0$ to get derivatives. Recall that energy per unit time is also power, and we can compare it with part (d):

$$P_{\text{aquired}} = \frac{1}{2}\frac{\mathrm{d}m}{\mathrm{d}t}v^2 = \frac{1}{2}P_{\text{ext}}.$$

Thus the energy aquired is $2.81/2 = 1.41$J per second.

f)

These two powers are not equal since friction is dissipating energy.