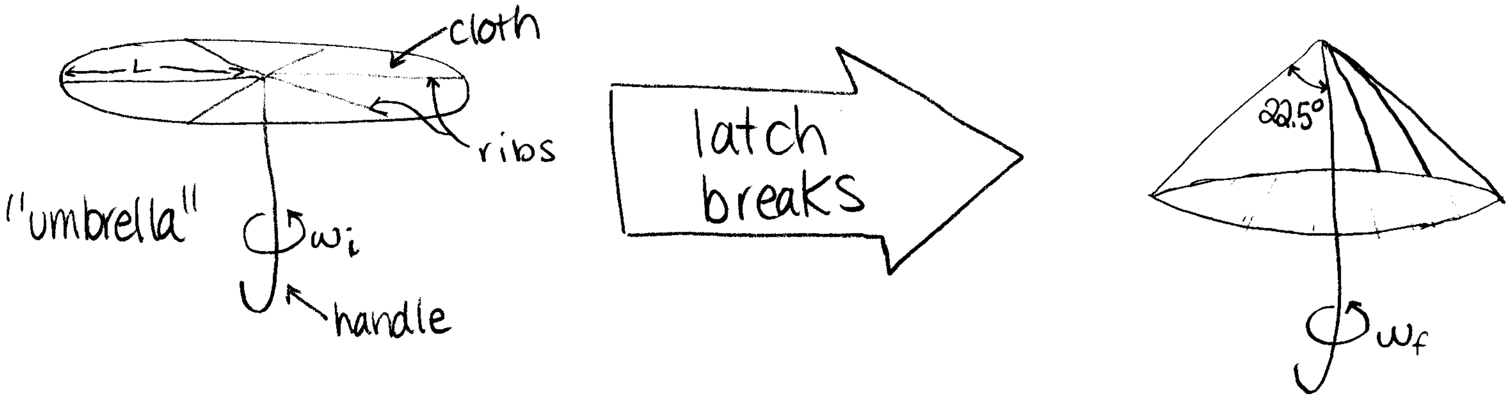
PHY180 Assignment 7 Solutions

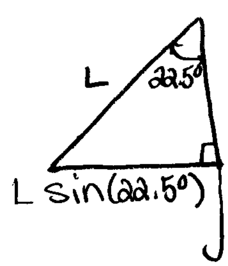
#1. Ch. 11, #32



- $\omega_1 = 1.25$ rad/s
- cloth & handle are negligible mass
- $\omega_f = ?$

Solution: Use conservation of angular momentum.

- the moment of inertia of one rod rotating about its end point is $\frac{1}{3}mL^2$
- take each rod to be length L, and take the sum of all their masses to be M.
- for one rod, after the latch breaks:



→ the effective length of the rod is $L\sin(22.5°)$.

- On to the solution:

$$L_i = L_f$$

$$I_i\omega_i = I_f\omega_f$$

$$\left(\sum_j \tfrac{1}{3}m_jL^2\right)\omega_i = \left(\sum_j \tfrac{1}{3}m_jL_f^2\right)\omega_f$$

$$\frac{1}{3}\left(\sum_j m_j\right)L^2\omega_i = \frac{1}{3}\left(\sum_j m_j\right)(L\sin(22.5°))^2\omega_f$$

$$\cancel{\frac{1}{3}}M\cancel{L^2}\omega_i = \frac{1}{3}ML^2\sin^2(22.5°)\omega_f$$

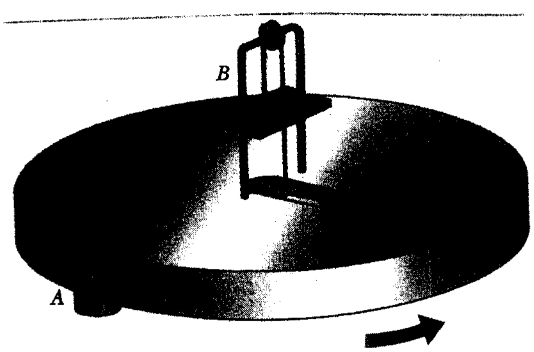
$$\omega_f = \frac{\omega_i}{\sin^2(22.5°)}$$

$$= \frac{1.25\text{ rad/s}}{\sin^2(22.5°)}$$

$$\boxed{\omega_f = 8.54\text{ rad/s}}$$

The final angular speed is 8.54 rad/s.

$\boxed{\#2.\ \text{Ch. } 11,\ \#46}$



disk: $M = 100\text{ kg}$
$R = 5.50\text{ m}$
$\omega = 2.50\text{ rev/s}$
$= 15.7\text{ rad/s}$

block: $m = 1.20\text{ kg}$
$v = 1.25\text{ cm/s}$
at $t = 0$ at centre of disk
at $t = 440\text{ s}$ at edge

Solution:

(a) find the torque the drive motor at A provides while the block is sliding.

- position of block: $r(t) = vt$ (constant speed)
- angular momentum of the block: $L = mr^2\omega$
  $= mv^2t^2\omega$

- to maintain a constant ω, the drive motor provides:

$$\tau = \frac{dL}{dt}$$

$$\tau = \frac{d}{dt}(mv^2t^2\omega)$$

$$= 2mv^2\omega t$$

$$= 2(1.20\,Kg)(0.0125\,m/s)^2(15.7\,rad/s)\,t$$

$$\boxed{\tau = (5.89\times10^{-3}\ W)\,t}$$

(b) torque at $t = 440S$.

$$\tau = (5.89\times10^{-3}\ W)(440S)$$

$$\boxed{\tau = 2.59\ Nm}$$

(c) find the power of the motor as a function of time.

$$P = \tau\omega$$

$$= (5.89\times10^{-3}\ W)t\,(15.7\ rad/s)$$

$$\boxed{P = (9.25\times10^{-2}\ W/s)\,t}$$

(d) power at $t = 440S$

$$P = (9.25\times10^{-2}\ W/s)(440S)$$

$$\boxed{P = 40.7\ W}$$

(e) find the string tension as a f'n of time:

$\Sigma F = 0$ (constant speed)

$$0 = T - \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r}$$

$$= \frac{m}{r}(\omega r)^2$$

$$= mr\omega^2$$

$$= mvt\omega^2$$

$$= (1.20\,\text{Kg})(0.0125\,\text{m/s})\,t\,(15.7\,\text{rad/s})^2$$

$$\boxed{T = (3.70\,\text{N/s})\,t}$$

(f) find the work done by the motor over the 440s interval:

$$W = \int_{t_i}^{t_f} P\,dt$$

$$= \int_{0s}^{440s} (9.25\times10^{-2}\,\text{W/s})\,t\,dt$$

$$= (9.25\times10^{-2}\,\text{W/s}) \int_{0s}^{440s} t\,dt$$

$$= (9.25\times10^{-2}\,\text{W/s}) \left(\frac{t^2}{2} \Big|_{0s}^{440s} \right)$$

$$= (9.25\times10^{-2}\,\text{W/s}) \left(\frac{(440s)^2}{2} - \frac{(0s)^2}{2} \right)$$

$$= 8.95\times10^{3}\,\text{Ws}$$

$$\boxed{W = 8.95\,\text{kJ}}$$

(g) find the work done on the string brake by the block.

• power given to the block: $P_b = \vec{F} \cdot \vec{v}$

$$\frac{dW_b}{dt} = Tv\cos 180°$$

$$\Rightarrow W_b = \int_{0s}^{440s} P_b \, dt$$

$$= \int_{0s}^{440s} Tv\cos 180° \, dt$$

$$= \int_{0s}^{440s} (3.70 \text{ N/s})t(0.0125 \text{ m/s})(-1)\, dt$$

$$= -(4.63\times10^{-2} \text{ W/s}) \int_{0s}^{440s} t\, dt.$$

$$= -(4.63\times10^{-2} \text{ W/s}) \left(\frac{t^2}{2} \Big|_{0s}^{440s} \right)$$

$$= -(4.63\times10^{-2} \text{ W/s}) \left(\frac{(440s)^2}{2} - \frac{(0s)^2}{2} \right)$$

$$= -4.48\times10^3 \text{ Ws}$$

$$\boxed{W_b = -4.48 \text{ KJ}}$$

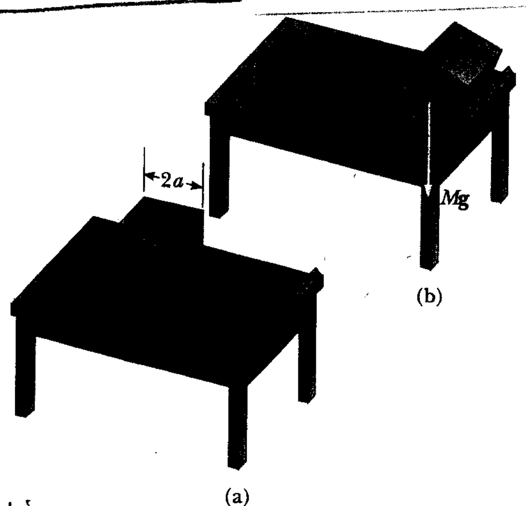
(h) find the total work done

$$W_T = W + W_b$$

$$= 8.95\text{KJ} - 4.48\text{KJ}$$

$$\boxed{W = 4.47 \text{ KJ}}$$

#3. Ch.11, #55



- find minimum v for cube to fall off table.
- $I = \frac{8Ma^2}{3}$
- hint: inelastic collision.

Solution:

- since it's an inelastic collision, only momentum is conserved. Use conservation of angular momentum:

$$L_i = L_f \quad \text{(during rotation)}$$

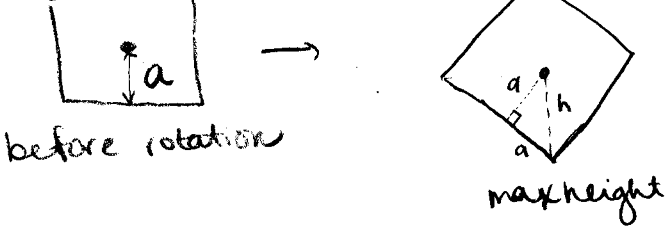
$$Mva = I\omega$$

$$\omega = \frac{Mva}{I}$$

$$= \cancel{M}v\cancel{a} \times \frac{3}{8\cancel{M}a^{\cancel{2}}}$$

$$\omega = \frac{3v}{8a}$$

- For the block to fall, its centre of mass must reach maximum height during rotation:



$$h^2 = a^2 + a^2$$

$$h = \sqrt{2a^2}$$

$$h = \sqrt{2}\,a.$$

- At the minimum speed, the block stops at this point.
  $\Rightarrow$ use conservation of energy.

$$E_{before} = E_{after}.$$

- Just as the block begins rotating:

$$E_{before} = \frac{1}{2}I\omega^2 + Mga$$ (set zero as height of tabletop.)

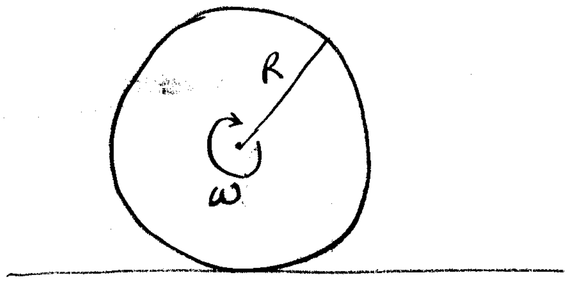
- When the block stops rotating:

$$E_{after} = Mgh$$
$$= Mg\sqrt{2}a$$

- Set these equal and solve for v:

$$\frac{1}{2}I\omega^2 + Mga = Mg\sqrt{2}a$$
$$\frac{1}{2}I\omega^2 = Mg(\sqrt{2}a - a)$$
$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\left(\frac{3v}{8a}\right)^2 = Mga(\sqrt{2}-1)$$
$$\frac{1}{2}\left(\frac{\cancel{8}\cancel{a^2}}{\cancel{3}}\right)\left(\frac{\cancel{9}^{3}v^2}{\cancel{64}_{8}\cancel{a^2}}\right) = ga(\sqrt{2}-1)$$
$$\frac{3v^2}{16} = ga(\sqrt{2}-1)$$
$$v^2 = \frac{16}{3}ga(\sqrt{2}-1)$$
$$\boxed{v = 4\sqrt{\frac{ga}{3}(\sqrt{2}-1)}}$$

#4. Ch. 11, #57



- coefficient of friction is μ.
- before touching surface, $\omega = \omega_i$.
- assume mass is M.

Solution:

(a) show time before pure rolling motion occurs is $R\omega_i / 3\mu g$

$$\Delta t = \frac{\Delta p}{F_F} \quad \text{(friction is unbalanced} \rightarrow \text{causes an impulse)}.$$

$$= \frac{\cancel{M}v}{\mu \cancel{M} g}$$

$$\Delta t = \frac{\omega R}{\mu g}$$

- To find ω, use conservation of angular momentum.

$$L_i = L_f$$

$$I\omega_i = I\omega + Mv$$

$$(\tfrac{1}{2}\cancel{M}R^2)\omega_i = (\tfrac{1}{2}\cancel{M}R^2)\omega + \cancel{M}(\omega R^2)$$

$$\tfrac{1}{2}\cancel{R^2}\omega_i = \tfrac{1}{2}\cancel{R^2}\omega + \omega \cancel{R^2}$$

$$\tfrac{1}{2}\omega_i = \tfrac{3}{2}\omega$$

$$\omega = \frac{\omega_i}{3}$$

- Subs. into above:

$$\Delta t = \frac{(\omega_i/3)R}{\mu g}$$

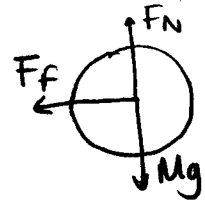
$$\boxed{\Delta t = \frac{R\omega_i}{3\mu g}}$$

(b) Show the distance travelled before pure rolling is $\frac{R^2\omega_i^2}{18\mu g}$

- Consider the centre of mass

$$V_i = 0$$

$$V_f = \omega R = \frac{\omega_i R}{3}.$$



- Newton's 2nd Law: $\Sigma F = ma$

$$F_f = Ma$$

$$\mu Mg = Ma$$

$$a = \mu g$$

$\therefore$ acceleration is constant.

- So we can use:

$$d = \tfrac{1}{2}(V_i + V_f)\Delta t$$

$$= \frac{1}{2}\left(0 + \frac{\omega_i R}{3}\right)\left(\frac{R\omega_i}{3\mu g}\right)$$

$$= \frac{1}{2}\,\frac{\omega_i R}{3}\,\frac{R\omega_i}{3\mu g}$$

$$\boxed{d = \frac{R^2\omega_i^2}{18\mu g}}$$