PHY 180
Assignment 6 Solutions
November 4, 2004

Assignment 6 Question 1

(a) For the bob to just barely swing through a circle, its speed will be zero at the top of the circle.

Let v_b be the initial speed of the bob after the collision. From Conservation of Energy:

$$K_i + U_i = K_f + U_f$$

$$\tfrac{1}{2} M v_b^2 + 0 = 0 + Mg2L$$

$$v_b^2 = 0 + 4gL$$

$$v_b = 2\sqrt{gL} \qquad (1)$$

For the bullet hitting the bob: From Conservation of Momentum,

$$p_i = p_f$$

$$mv + 0 = m\left(\frac{v}{2}\right) + Mv_b \qquad (2)$$

Using (1) in (2)

$$mv - \tfrac{1}{2} mv = M\, 2\sqrt{gL}$$

$$\tfrac{1}{2} mv = 2\, M\sqrt{gL}$$

$$v = 4\frac{M}{m}\sqrt{gL}$$

(b) For the bob to just barely swing through a circle, its speed will be zero at the top of the circle.

Let v_b be the initial speed of the bob after the collision. Work is done by the force of gravity to reduce the kinetic energy of the bob to zero.

$$\text{Work Done} = W = \int_0^{\pi} \tau \, d\theta$$

$$= \int_0^{\pi} \vec{r} \times \vec{F} \, d\theta$$

$$= LMg\int_0^{\pi} \sin\theta \, d\theta = LMg\left[-\cos\theta\right]\Big|_0^{\pi} = -LMg[-1-1]$$

$$W = 2LMg$$

Kinetic energy at the bottom is $\tfrac{1}{2} M v_b^2$.

$$\therefore \qquad \tfrac{1}{2} M v_b^2 = 2LMg \qquad (1)$$

From Conservation of Angular Momentum

$$mLv = \tfrac{1}{2} mLv + MLv_b$$

$$Mv_b = \tfrac{1}{2} mv$$

$$v_b = \tfrac{1}{2}\frac{m}{M} v \qquad (2)$$

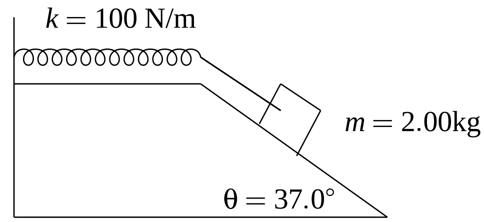
Using (2) in (1)

$$\tfrac{1}{2} M \, \tfrac{1}{4} \frac{m^2}{M^2} v^2 = 2LMg$$

$$v^2 = 16 Lg \, \frac{M^2}{m^2}$$

$$v = 4\frac{M}{m}\sqrt{gL}$$

Question 2: Chapter 8, #54



Use conservation of energy:

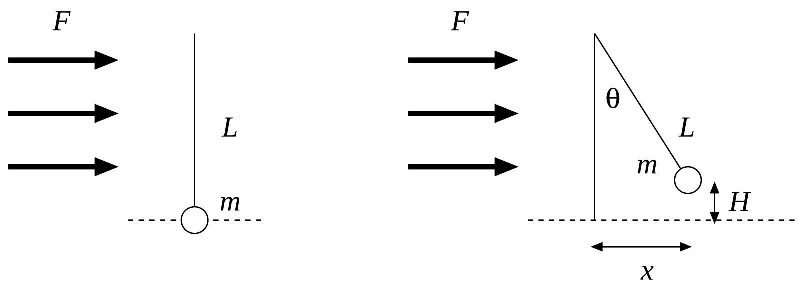
$$\begin{aligned} K_i + U_i - F_f d &= K_f + U_f \\ 0 + mgh_i - F_f d &= mgh_f + \frac{1}{2}kd^2 \end{aligned}$$

where h_i and h_f are the initial and final heights of the block, d is the amount by which the block moves down the incline ($d = 20.0$ cm), i.e. the amount by which the spring is stretched, and F_f is the force of friction.

Using $F_f = \mu N$ we get,

$$\begin{aligned} -\mu N d &= -mg(h_i - h_f) + \frac{1}{2}kd^2 \\ \mu(mg\cos\theta)d &= mg(d\sin\theta) - \frac{1}{2}kd^2 \\ \mu &= \frac{mg(d\sin\theta) - \frac{1}{2}kd^2}{(mg\cos\theta)d} \\ \mu &= \frac{(2.00)(9.80)(0.200)(\sin 37.0°) - \frac{1}{2}(100)(0.200)^2}{(2.00)(9.80)\cos 37.0°(0.200)} \\ \mu &= 0.114 \end{aligned}$$

Question 3: Chapter 8, #69



(a)

First find x in terms of H and L:

$$\begin{aligned}(L-H)^2+x^2 &= L^2\\ L^2-2LH+H^2+x^2 &= L^2\\ x^2 &= 2LH-H^2\end{aligned}$$

Now use conservation of energy:

$$\begin{aligned}K_i+U_i+Fx &= K_f+U_f\\ 0+0+Fx &= mgH\\ F^2x^2 &= (mgH)^2\\ F^2(2LH-H^2) &= (mgH)^2\\ H^2(F^2+(mg)^2)-H(F^22L) &= 0\\ H(H(F^2+(mg)^2)-F^22L) &= 0\end{aligned}$$

Therefore, $H=0$ or, as the upper limit,

$$\begin{aligned}H(F^2+(mg)^2)-F^22L &= 0\\ H &= \frac{F^22L}{F^2+(mg)^2}\\ H &= \frac{2L}{1+(mg/F)^2}\end{aligned}$$

For $H\to 0$, need $F\to 0$, i.e. no wind gives a maximum height of zero.

For $H\to 2L$, need $F\to\infty$, i.e. a very strong wind causes the ball to blow all the way around.

So, the equation is valid in for both the cases $0\le H\le L$ and $L\le H\le 2L$.

(b)

$$
\begin{aligned}
H &= \frac{2(2.00)}{1+[(2.00)(9.80)/(14.7)]^2} \\
H &= 1.44\,\mathrm{m}
\end{aligned}
$$

(c)

Take sum of forces in x-dir:

$$
\begin{aligned}
F - T\sin\theta &= 0 \\
T &= \frac{F}{\sin\theta}
\end{aligned}
$$

where T is the tension in the string.

Now in the y-dir:

$$
\begin{aligned}
T\cos\theta - mg &= 0 \\
\frac{F}{\sin\theta}\cos\theta - mg &= 0 \\
F\tan\theta &= mg \\
\tan\theta &= \frac{F}{mg} \\
\theta &= 36.9^\circ
\end{aligned}
$$

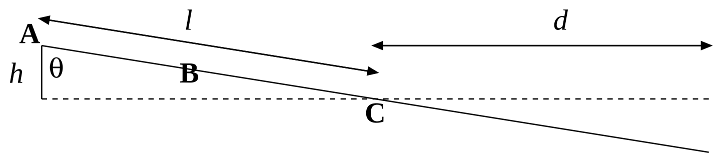
From geometry, we have

$$
\begin{aligned}
\cos\theta &= \frac{L - H_{eq}}{L} \\
H_{eq} &= L(1-\cos\theta) \\
H_{eq} &= (2.00)(1-\cos 36.9^\circ) \\
H_{eq} &= 0.400\,\mathrm{m}
\end{aligned}
$$

(d)

At most the equilibrium height can be equal to L. This occurs when $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90^\circ$ and thus $H_{eq} \to L$.

Question 4: Chapter 8, #74



(a)

Can use kinematics equations for constant acceleration or conservation of energy, lets use conservation of energy between point **A** and **C**:

$$
\begin{aligned}
K_A + U_A &= K_C + U_C \\
\frac{1}{2}mv_A^2 + mgh &= \frac{1}{2}mv_C^2 \\
v_C &= \sqrt{v_A^2 + 2gh} \\
v_C &= \sqrt{(2.50)^2 + 2(9.80)(9.76)} \\
v_C &= 14.1\,\mathrm{ms}^{-1}
\end{aligned}
$$

(b)

The work done by the friction of the water is equal to the change in kinetic energy,

$$
\begin{aligned}
W_{\text{water}} &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_C^2 \\
W_{\text{water}} &= 0 - \frac{1}{2}mv_C^2 \\
W_{\text{water}} &= -\frac{1}{2}(80.0)(14.1)^2 \\
W_{\text{water}} &= -7.90 \times 10^3\,\mathrm{J}
\end{aligned}
$$

(c)

$$
\begin{aligned}
W_{\text{water}} &= -F_{\text{water}}d \\
F_{\text{water}} &= -\frac{W_{\text{water}}}{d} \\
F_{\text{water}} &= \frac{7.90 \times 10^3}{50.0} \\
F_{\text{water}} &= 158\mathrm{N}
\end{aligned}
$$

(d)

Normal force at point **B** is,

$$
\begin{aligned}
N_B &= mg\sin\theta \\
N_B &= mg\left(\frac{\sqrt{l^2-h^2}}{l}\right) \\
N_B &= (80.0)(9.80)\left(\frac{\sqrt{(54.3)^2-(9.76)^2}}{54.3}\right) \\
N_B &= 771\text{N}
\end{aligned}
$$

(e)

Summing up the forces acting on the sled at point **C** (keeping in mind in ramp curves) we have:

$$
\begin{aligned}
N_C - mg &= \frac{mv_C^2}{R} \\
N_C &= mg + \frac{mv_C^2}{R} \\
N_C &= (80.0)(9.80) + \frac{(80.0)(14.1)^2}{20.0} \\
N_C &= 1.57\times 10^3\text{N}
\end{aligned}
$$