Solution to (1) of Assignment 3 2004

$s = 30.0 \pm 0.2$ cm
$s' = 20.0 \pm 0.1$ cm

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \text{ or } f = \frac{s\,s'}{s+s'} = \frac{600}{50} = 12 \text{ cm}$$

To find the error in f we use the formula in the form $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$ because the right hand side of the equation is a sum of terms of two **independent variables**. It is for this reason that we do not use the form $f = \frac{s\,s'}{s+s'}$ since the numerator and denominator of the quotient on the right side of the equation are not independent.

We use the rule for a sum of terms.

$$\Delta\left(\frac{1}{f}\right) = \sqrt{\Delta\left(\frac{1}{s}\right)^2 + \Delta\left(\frac{1}{s'}\right)^2} \quad (1)$$

and using the rule for a quotient $\Delta\left(\frac{1}{f}\right) = \frac{\Delta f}{f^2}$ and similarly $\Delta\left(\frac{1}{s}\right) = \frac{\Delta s}{s^2}$ and .

Using these results in (1) gives $\Delta f = f^2\sqrt{\left(\frac{\Delta s}{s^2}\right)^2 + \left(\frac{\Delta s'}{s'}\right)^2} = 12^2\sqrt{\left(\frac{0.2}{30.0^2}\right)^2 + \left(\frac{0.1}{20.0^2}\right)^2}$

Which gives $\Delta f = 0.0482$

Thus $f = 12.0 \pm 0.05$ cm.

PHY180F ASSIGNMENT 3 (2004)

/ Solutions /

Problem # 1 $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$; $s = 30.0 \pm 0.2$ [cm]
$s' = 20.0 \pm 0.1$ [cm]

and solving for f we get

$$f = \frac{ss'}{(s+s')} \Rightarrow f = \frac{30.0 \times 20.0}{(30.0 + 20.0)} = 12.0 \text{ [cm]}$$

and the associated uncertainty Δf is:

$$\Delta f = \sqrt{\left[\left(\frac{\partial f}{\partial s}\right) \cdot \Delta s\right]^2 + \left[\left(\frac{\partial f}{\partial s'}\right) \cdot \Delta s'\right]^2}$$

$$\frac{\partial f}{\partial s} = \frac{s'(s+s') - ss'}{(s+s')^2} = \frac{s's + s'^2 - ss'}{(s+s')^2} = \frac{s'^2}{(s+s')^2} = \frac{(20.0)^2}{(30.0+20.0)^2} = 0.16$$

$$\frac{\partial f}{\partial s'} = \frac{s(s+s') - ss'}{(s+s')^2} = \frac{s^2 + ss' - ss'}{(s+s')^2} = \frac{s^2}{(s+s')^2} = \frac{(30.0)^2}{(30.0+20.0)^2} = 0.36$$

Therefore

$$\Delta f = \sqrt{[0.16 \times 0.2]^2 + [0.36 \times 0.1]^2} =$$

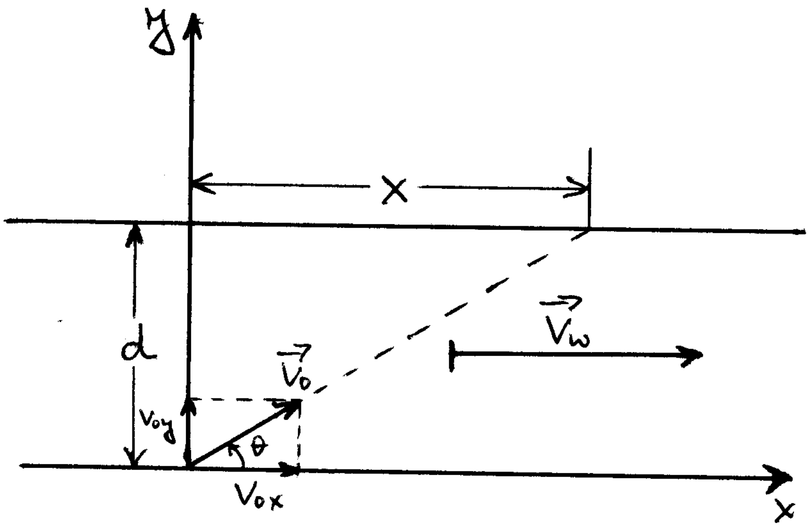
$$= \sqrt{(0.032)^2 + (0.036)^2} =$$

$$= \sqrt{0.001024 + 0.001296} = 0.0481664$$

Approximating to the second digit after the decimal point

$$\boxed{f \pm \Delta f = 12.0 \pm 0.05 \text{ [cm]}}$$

Problem #2



$|\vec{V}_w| = v_w = 2.50$ [m/s]

$d = 80.0$ [m]

$|\vec{V}_o| = v_o = 1.50$ [m/s]

Total speed (velocity!)

$$\vec{V} = \vec{V}_w + \vec{V}_o$$

$$\therefore \begin{aligned} x &: V_x = V_o\cos\theta + V_w \\ y &: V_y = V_o\sin\theta \end{aligned} \Rightarrow \left[\begin{aligned} x &= (V_o\cos\theta + V_w)\cdot t \\ d &= V_o\sin\theta\cdot t \end{aligned}\right.$$

(a) $t_{min} \Rightarrow \max|V_o\sin\theta| \rightarrow \sin\theta \equiv 1 \Rightarrow \theta = \frac{\pi}{2}$

(b) $x = (V_o\cos\frac{\pi}{2} + V_w)\cdot t = V_w\cdot t = \left(\frac{V_w}{V_o}\right)d = 133.3$ [m]

(c) $x = (V_w + V_o\cos\theta)\cdot t = (V_w + V_o\cos\theta)\left(\frac{d}{V_o\sin\theta}\right)$

$$\therefore x = x(\theta) = d\left[\left(\frac{V_w}{V_o}\right)\frac{1}{\sin\theta} + \left(\frac{\cos\theta}{\sin\theta}\right)\right]$$

$x_{min} \Rightarrow \left(\frac{\partial x}{\partial\theta}\right) = 0$ which leads to

$$\frac{\partial x}{\partial\theta} = d\left[\left(\frac{V_w}{V_o}\right)\left(-\frac{\cos\theta}{\sin^2\theta}\right) + \frac{\overbrace{(-\sin^2\theta - \cos^2\theta)}^{-1}}{\sin^2\theta}\right] = 0$$

so that

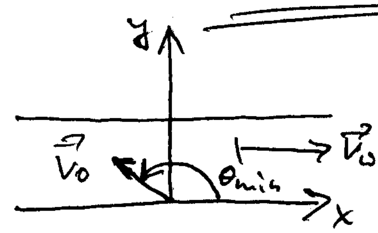
$$\frac{\partial x}{\partial \theta} = -d\left[\left(\frac{V_w}{V_o}\right)\cos\theta + 1\right]/\sin^2\theta = 0$$

Assuming that $\sin\theta \neq 0 \Rightarrow \theta \neq 0, \pi$ we get

$$\left(\frac{V_w}{V_o}\right)\cos\theta + 1 \equiv 0 \Rightarrow \cos\theta = -\left(\frac{V_o}{V_w}\right)$$

$$\therefore \quad \theta = \arccos\left[-\left(\frac{V_o}{V_w}\right)\right] = \arccos\left[-\left(\frac{1.50}{2.50}\right)\right] \cong 126.87^\circ$$

$$X_{min} \rightarrow \boxed{\theta_{min} \cong 126.87^\circ}$$



(d) $$x = (V_w + V_o\cos(126.87^\circ))\left(\frac{d}{V_o\sin(126.87^\circ)}\right) =$$

$$= d\left[\left(\frac{V_w}{V_o}\right)\frac{1}{\sin(126.87^\circ)} + \frac{\cos(126.87^\circ)}{\sin(126.87^\circ)}\right] =$$

$$= 80.0\left[\left(\frac{2.50}{1.50}\right)\times 1.25 + (-0.6)\times 1.25\right] =$$

$$= 80.0\,[2.08 - 0.75] = 80.0 \times 1.33$$

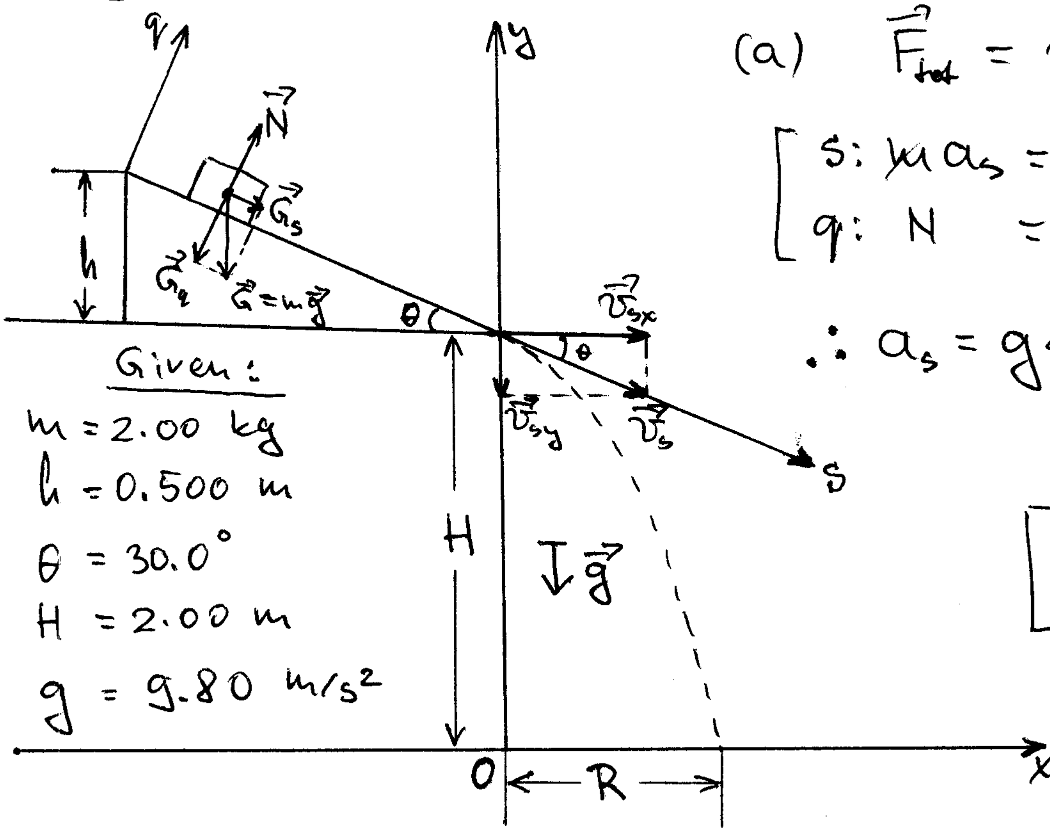
$$\therefore \quad \boxed{X_{min} = 106.4\ [m]} < 133.3\ [m]$$

Conclusion:

$$\boxed{\begin{array}{l} t_{min} = \dfrac{d}{V_o} = \dfrac{80.0}{1.50} = 53.3\ [s] \text{ at } \theta = \dfrac{\pi}{2} \\ X_{min} = 106.4\ [m] \text{ at } \theta = 126.87^\circ \end{array}}$$

Problem #3

The incline is frictionless!



Given:

$m = 2.00$ kg

$h = 0.500$ m

$\theta = 30.0°$

$H = 2.00$ m

$g = 9.80$ m/s²

(a) $\vec{F}_{tot} = m\vec{a}$

$$\begin{cases} s: \cancel{m} a_s = \cancel{m} g \sin\theta \\ q: N = mg\cos\theta \end{cases}$$

$$\therefore a_s = g\sin\theta = \frac{1}{2}g = \underline{\underline{4.9 \text{ m/s}^2}}$$

$$\boxed{a_s = 4.9 \text{ m/s}^2}$$

(b)
$$\begin{cases} v_s = \cancel{v_{s0}}^{\,0} + a_s t_1 \longrightarrow t_1 = \dfrac{v_s}{a_s} \\ s = \cancel{s_0}^{\,0} + \cancel{v_{s0}}^{\,0} t_1 + \frac{1}{2} a_s t_1^2 \Rightarrow s = \frac{1}{2} a_s \left(\dfrac{v_s^2}{a_s^2}\right) = \dfrac{1}{2}\dfrac{v_s^2}{a_s} \end{cases}$$

$$\therefore v_s^2 = 2 a_s s \Rightarrow v_s = \sqrt{2 a_s s} \; ; \; s = \frac{h}{\sin\theta}$$

and $$v_s = \sqrt{\cancel{2}\cdot\frac{1}{\cancel{2}}g\cdot\frac{h}{\sin\theta}} = \sqrt{\frac{gh}{\sin\theta}} = \sqrt{2gh} = \boxed{3.13 \text{ m/s}}$$

(c)
$$\begin{cases} R = v_{sx}\cdot t_2 \longrightarrow t_2 = \dfrac{R}{v_{sx}} \\ 0 = H - v_{sy}\cdot t_2 - \frac{1}{2} g t_2^2 \Rightarrow \frac{1}{2} g \left(\dfrac{R}{v_{sx}}\right)^2 + v_{sy}\left(\dfrac{R}{v_{sx}}\right) - H = 0 \end{cases}$$

where
$$v_{sx} = v_s\cos\theta = \frac{\sqrt{3}}{2} v_s = \sqrt{\frac{3gh}{2}}$$
$$v_{sy} = v_s\sin\theta = \frac{1}{2} v_s = \sqrt{\frac{gh}{2}}$$

so that $$\frac{v_{sy}}{v_{sx}} = \frac{\frac{1}{2}v_s}{\frac{\sqrt{3}}{2}v_s} = \frac{1}{\sqrt{3}}$$

Therefore $$\frac{1}{\cancel{2}}\cancel{g}\frac{R^2}{\left(\frac{3\cancel{g}h}{\cancel{2}}\right)} + \frac{1}{\sqrt{3}}R - H = 0$$

$$\therefore \underbrace{R^2 + \sqrt{3}hR - 3hH = 0}$$

$$R_{1,2} = \frac{-\sqrt{3}h \oplus \sqrt{3h^2 + 4\cdot 3hH}}{2}$$

or $$R = \frac{\sqrt{3}}{2}h\left\{\sqrt{1+4\left(\frac{H}{h}\right)} - 1\right\} = \underline{\underline{\frac{\sqrt{3}}{2}h\left\{\sqrt{17}-1\right\}}}$$

$$\therefore R = \frac{\sqrt{3}}{2}h\{3.1231\} = 1.352\,m$$

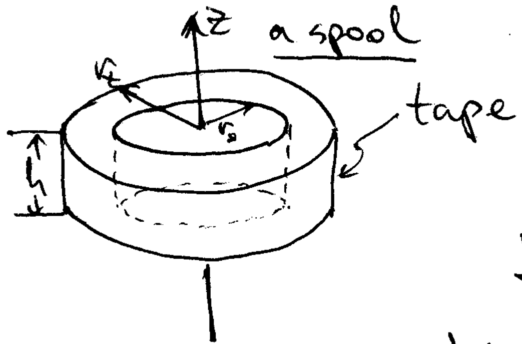
$$\boxed{R = 1.352\,m}$$

(d) $$t = t_1 + t_2 = \frac{v_s}{a_s} + \frac{R}{v_{sx}} = 0.639 + 0.499 \triangleq 1.138\,s$$

$$\therefore \boxed{t_{tot} \cong 1.138\ s}$$

(e) The mass of the block does not affect any of the above calculations, since it cancells out from Newton's Law. The whole treatment is purely kinematic!

Problem #4



We will use the principle of conservation of mass! Let M be the mass of the whole tape when it is totally wound on the first spool. Then

h-width of tape!, $M = \rho_{tape} \text{Volume} = \rho_t h \pi (r_t^2 - r_s^2)$

In any given moment in time the total mass of the tape should be the same (i.e.)

$$M = m_1 + m_2$$

where m_1 - mass of tape on spool 1
and m_2 - mass of tape on spool 2

$$\therefore \cancel{\rho_t h \pi}(r_t^2 - r_s^2) = \cancel{\rho_t h \pi}(r^2 - r_s^2) + \cancel{\rho_t h \pi}(r_2^2 - r_s^2)$$

$$r_t^2 - \cancel{r_s^2} = r^2 - \cancel{r_s^2} + r_2^2 - r_s^2$$

or $$\boxed{r_2^2 = r_s^2 + r_t^2 - r^2 \Rightarrow r_2 = \sqrt{r_s^2 + r_t^2 - r^2}}$$

and since $v = \text{const.} = \omega_1 r = \omega_2 r_2$

$$\therefore \boxed{\omega_1 = v/r \quad \text{and} \quad \omega_2 = v/(r_s^2 + r_t^2 - r^2)^{1/2}}$$ QED

(b) $\omega_1(\max) \longrightarrow \omega_1^{max} = \frac{v}{r_s}$; $\omega_2^{min} = \frac{v}{r_t}$; $r_t > r_s$

when $r \to r_s$ and $\cancel{r_s^2} + r_t^2 - \cancel{r_s^2} = r_t^2$