ASSIGNMENT 2 SOLUTIONS (2004)

1) Exercise 5.1

1) No, the width of the curve as measured by the standard deviation does not depend on the number of times the measurement is repeated.

2) Mean = 49
Standard deviation = 7

3) The width of this spread in values decreases with the number of repeated measurements.

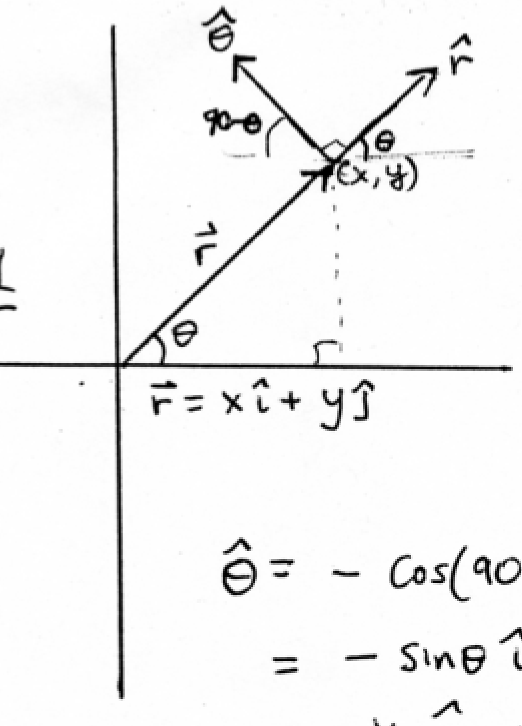
4) Standard deviation = the square root of the mean

5) Statistical uncertainty in the number of goals he has scored = $\sqrt{16} = 4$.

2)

$\cos\theta = \dfrac{x}{r}$

$\sin\theta = \dfrac{y}{r}$



a) $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{x\hat{i} + y\hat{j}}{\sqrt{x^2+y^2}}$

or $\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$= \dfrac{x}{r}\hat{i} + \dfrac{y}{r}\hat{j}$

$= \dfrac{x\hat{i} + y\hat{j}}{\sqrt{x^2+y^2}}$

$\hat{\theta} = -\cos(90-\theta)\,\hat{i} + \cos\theta\,\hat{j}$

$= -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

$= -\dfrac{y}{r}\hat{i} + \dfrac{x}{r}\hat{j}$

$= \dfrac{-y\hat{i} + x\hat{j}}{\sqrt{x^2+y^2}}$

2 b) $A = \sin\theta\,\hat{r} - r\cos\theta\,\hat{\theta}$

$$= \frac{y}{\sqrt{x^2+y^2}}\left(\frac{x\hat{\imath} + y\hat{\jmath}}{\sqrt{x^2+y^2}}\right) - \left(\sqrt{x^2+y^2}\right)\frac{x}{\sqrt{x^2+y^2}}\left(\frac{-y\hat{\imath} + x\hat{\jmath}}{\sqrt{x^2+y^2}}\right)$$

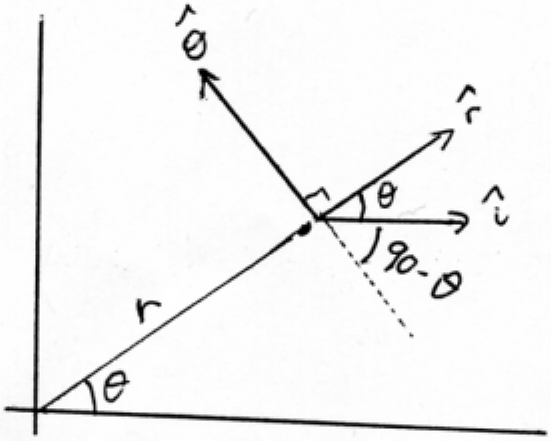
$$= \frac{yx\hat{\imath}}{x^2+y^2} + \frac{y^2\hat{\jmath}}{x^2+y^2} + \frac{xy\hat{\imath}}{\sqrt{x^2+y^2}} - \frac{x^2\hat{\jmath}}{\sqrt{x^2+y^2}}$$

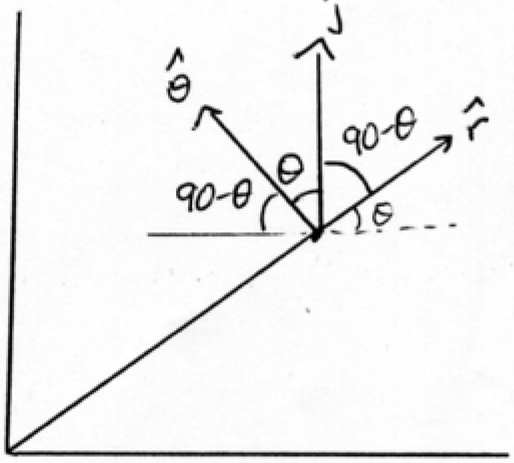
$$= \left(\frac{xy}{x^2+y^2} + \frac{xy}{\sqrt{x^2+y^2}}\right)\hat{\imath} + \left(\frac{y^2}{x^2+y^2} - \frac{x^2}{\sqrt{x^2+y^2}}\right)\hat{\jmath}$$

c)

$\frac{x}{r} = \cos\theta$

$\frac{y}{r} = \sin\theta$





$$\hat{\imath} = \cos\theta\,\hat{r} - \cos(90-\theta)\,\hat{\theta}$$
$$= \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

$$\hat{\jmath} = \cos(90-\theta)\,\hat{r} + \cos\theta\,\hat{\theta}$$
$$= \sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}$$

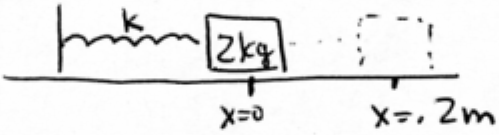
d) $\vec{B} = xy^2\hat{\imath} + x^2y\hat{\jmath}$

$$= (r\cos\theta)(r\sin\theta)^2\left[\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\right] + (r\cos\theta)^2(r\sin\theta)\left[\sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}\right]$$

$$= r^3\cos^2\theta\sin^2\theta\,\hat{r} - r^3\cos\theta\sin^3\theta\,\hat{\theta}$$
$$+ r^3\cos^2\theta\sin^2\theta\,\hat{r} + r^3\sin\theta\cos^3\theta\,\hat{\theta}$$

$$= r^3\left\{2\sin^2\theta\cos^2\theta\,\hat{r} + (\sin\theta\cos^3\theta - \cos\theta\sin^3\theta)\,\hat{\theta}\right\}$$

Chapter 15 #20



a) $k = F/x = \frac{20N}{0.2m} = \boxed{100\ N/m}$

b) $\omega = \sqrt{k/m} = \sqrt{50}$ rad/s

$f = \frac{\omega}{2\pi} = \boxed{1.13\ Hz.}$

c) $V_{max} = A\omega = (0.2)(\sqrt{50}) = \boxed{1.41\ m/s}$ V is maximum when $X = 0$.

d) $a_{max} = A\omega^2 = (0.2)(50) = \boxed{10\ m/s^2}$ a is max at $X = \pm A$

e) $E_{total} = E_{k\,max} = \frac{1}{2} m V_{max}^2 = \boxed{2\ J}$

f) From energy conservation: $E_{tot} = \frac{1}{2} mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$\Rightarrow V = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega\sqrt{A^2 - x^2}$

$\Rightarrow$ for $x = \frac{1}{3}A$, $v = \omega\sqrt{A^2 - \frac{1}{9}A^2} = \omega\sqrt{\frac{8}{9}A^2} = \boxed{1.33\ m/s}$

g) $a = \omega^2 x = 50\left(\frac{.2}{3}\right) = \boxed{3.33\ m/s^2}$

ch15 # 52

a) Use conservation of Energy.

total P.E. stored in compressed spring $= \frac{1}{2}kA^2$ = total initial energy of the system

When the masses pass through $x=0$, $PE = 0$ & but they are stuck together so $v_1 = v_2$

$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$\Rightarrow E_{tot} = \frac{1}{2}kA^2 = \frac{1}{2}(m_1+m_2)v^2 \rightarrow v = \sqrt{\frac{kA^2}{m_1+m_2}} = \boxed{0.5\ m/s}$

b) Using conservation of energy for the m_1-spring system alone:

at $x=0$, the mass has a speed of $v = .5\ m/s$.

$\Rightarrow KE_{(m_1\text{-spring})} = \frac{1}{2}m_1v^2 = \frac{1}{2}(9)(.5)^2 = 1.125\ J$.

$KE_{max} = PE_{max}$ of the m_1-spring system $\Rightarrow 1.125\ J = \frac{1}{2}k(A')^2$

where A' = amplitude of the m_1-spring system. $\rightarrow A' = \sqrt{\frac{2(1.125)}{k}} = 0.15\ m$

Period of m_1-spring system $= T = 2\pi\sqrt{\frac{m_1}{k}} = 1.885\ s.$ $\Rightarrow$ time at which it becomes fully stretched for the first time is $t = \frac{T}{4} = .471\ s.$ after passing through $x=0$.

$\Rightarrow$ distance between m_1 & m_2 is $D = v(T/4) - A' = \boxed{8.56\ cm}$