UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, DECEMBER 2003

First Year - Engineering Science

PHY180F - PHYSICS I - MECHANICS

Exam Type: C

Examiner: J. M. Pitre

Duration 2½ hours

Do all questions. All questions are of equal value. There are 80 possible marks.

Start the answer to each question on a new page.

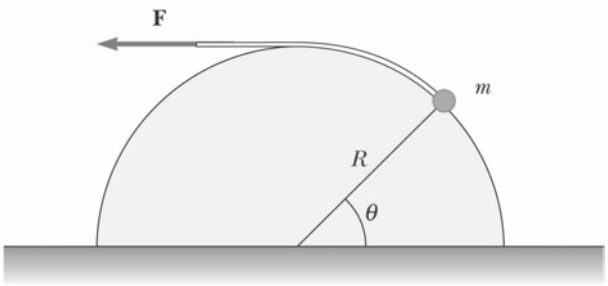
You do not need to do the problems in order.

Use g = 9.80 ms^{-2} for the acceleration due to gravity in all problems.

1) A football is thrown straight toward a receiver with an initial speed of 20.0 ms^{-1}, at an angle of 30.0° above the horizontal. At that instant, the receiver is 20.0 m from the person who threw the football. In what direction and with what constant speed should the receiver run in order to catch the football at the height at which it was thrown?

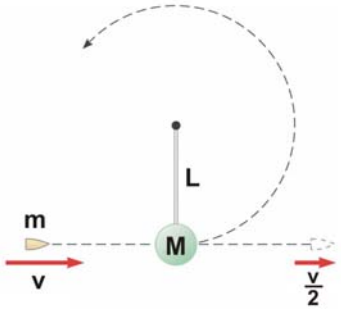
2) A particle undergoes uniform circular motion with a period **T** in a circle of radius **r**.
   a) For a point in the first quadrant, express the unit vectors $\hat{r}$ and $\hat{\theta}$ (cylindrical or polar coordinates) in terms of $\hat{i}$ and $\hat{j}$ (Cartesian coordinates) and the angle θ.
   b) Write down the speed in terms of the variables given in the problem.
   c) Write down the expression (equation) which gives θ as a function of time.
   d) Write down the position vector $\vec{r}$ of the particle in cylindrical coordinates.
   e) **Derive** an expression for $\vec{v}$ in cylindrical coordinates showing all your work.

3) A small particle of mass **m** is pulled to the top of a frictionless half-cylinder of radius **R** by a cord that passes over the top of the cylinder as shown in the diagram.



   a) If the particle moves at a constant speed, find the magnitude of the force $\vec{F}$. Use a diagram, explain your reasoning and show your work.
   b) Write down $\vec{F}$ using cylindrical coordinates and unit vectors.
   c) Without using the concept of potential energy, calculate $W = \int \vec{F} \cdot d\vec{r}$, the work done in moving the particle at constant speed along the circular path from the bottom to the top of the half-cylinder. Use cylindrical coordinates and unit vectors for all quantities.

4) A bullet of mass **m** and initial speed **v** passes completely through a pendulum bob of mass **M**. The bullet emerges with a speed ½ **v**. The pendulum bob is suspended by a stiff rod of length **L** and negligible mass. What is the minimum value of **v** such that the pendulum bob will barely swing through a complete vertical circle? State clearly the justification for any statements you make or equations that you write down.



5) A body of mass **m** is sliding from left to right in the +**x** direction on a frictionless surface at a speed $\mathbf{v_o}$. At **x** = 0 and **t** = 0 the surface changes in such a way that the magnitude of the retarding (or slowing down) force on the mass is equal to one half the magnitude of the speed. Derive an equation which gives the speed **v** as a function of time.

6) One of Kepler's laws (as stated in your text) is "The [BLANK 1] of any planet is proportional to the [BLANK 2] of the semi-major axis of the elliptical orbit."
   a) What is the phrase which replaces BLANK 1?
   b) What is the word which replaces BLANK 2?
   c) For a circular orbit derive this law, defining variables that you use and explaining all your steps.

7) A thin rod of length **L** and mass **M** is suspended freely from one end. It is pulled aside and swung about a horizontal axis and when the centre of mass passes through its lowest point the speed of the centre of mass is 1.0 ms^{-1}. Neglect friction and air resistance. State and justify any assumptions that you may have to make when answering this question.
   a) How high does the centre of mass rise from its lowest position?
   b) In terms of **L** and **g**, how long does it take for the centre of mass to rise from its lowest position to its highest position?

8) A steel wire of length 2.0 m is placed over a pulley of mass ½**M**. (A pulley is a disk or solid cylinder.) Two masses, **M** and 2**M** are attached to the ends of the wire as shown in the diagram. The masses are allowed to move and when they pass, at the same height, the wire between the mass **M** and the pulley has been stretched by 5.0 x 10^{-4} m. At that instant, by how much has the wire between the 2**M** mass and the pulley been stretched?