THE WILBERFORCE SPRING

REFERENCES

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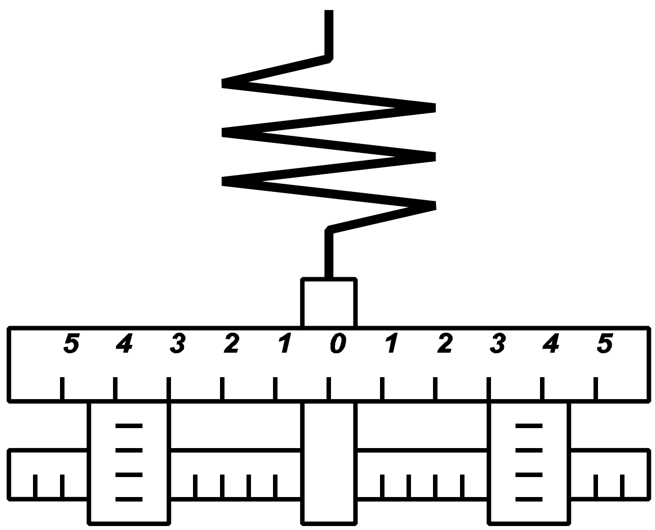

INTRODUCTION

The Wilberforce spring can undergo two types of harmonic motion. First, it can oscillate up and down, in which case the period should depend only on the particular spring used and the total mass on the end of the spring. Note that the simple relation for the mass on the end of a massless spring that you learn in lectures is not applicable in this case: *i.e.*,

$$T_{transl} \neq 2\pi \sqrt{\frac{M}{k}}$$

where M = mass of the bob, and k = spring constant (force/extension), because in this situation you cannot neglect the fact that the centre of mass of the spring is also oscillating. In fact, for the uniform massy spring, as shown in the above references:

$$T_{transl} = 2\pi \sqrt{\frac{M + \frac{m}{3}}{k}}$$

where m = mass of the spring

The other type of harmonic motion possible is a rotational oscillation. The period of the rotational oscillation depends on the particular spring and on the moment of inertia I of the mass (weight plus frame system) on the end of the spring. I can be varied by moving the position of the movable weights. The total moment of inertia for the weights plus frame is $I = I_o + 2m_w d^2$, where I_o = moment of inertia of frame(which can be measured using the methods of, e.g.the experiment on the Torsion Pendulum, *q.v.*), m_w = mass of one movable weight, and d = distance of each of the movable weights from the centre. For the massy spring with the frame and weights attached to the end,

$$T_{rot} = 2\pi \sqrt{\frac{I + \alpha I_1}{c}}$$

where I_1 = moment of inertia of the spring, α = a constant (representing the "fraction" of the spring that is oscillating), and c = torsion constant of the spring (torque/angular displacement). (Theoretical expressions for these constants in terms of the parameters of the spring can be found in the quoted references.)

In this system, these two types of harmonic motion, translational and rotational, are not entirely independent; there is a slight coupling between them. This is because the spring has a slight tendency to coil and uncoil as it is extended or compressed. The Wilberforce spring is thus an example of two weakly coupled resonant systems, other examples being the splitting of energy levels in the ammonia molecule, two simple pendula of similar length with a spring jointing the upper parts of their strings or thus, the Wilberforce spring is a good way to study mechanical resonance in coupled systems.

THE EXPERIMENT

PART I:

In order to study the mechanical resonance you will want to study the transfer of energy from one mode of oscillation to the other as a function of the period of the rotational motion, particularly for values of T_{rot} close to T_{transl}. (Note that the apparatus allows you to vary T_{rot}, but T_{transl} is fixed.) The energy of the translational motion can be estimated from the maximum amplitude in case of the oscillations, and the rotational energy from the maximum angular amplitude (visual estimates only). Remember that the energy varies with the *square* of the amplitude.

Another interesting parameter is the time necessary to complete the maximum energy transfer from one mode to the other as a function of T_{rot}.

For $T_{rot} = T_{transl}$ the phase relationship between the two modes of oscillation may be examined. Some springs tend to coil up as they are being extended, others uncoil. As shown in the references quoted above, the relative value of the shear modulus to Young's modulus for the metal of the spring determines which case holds for your spring. This can also be determined experimentally by watching the spring as it transfers energy back and forth. Also, you may want to watch the phase relationship between the two modes when energy is being transferred in one direction versus the phase when the energy transfer is going the other way.

PART II:

There are various additional investigations:

(a) Data you have already obtained above can be used to verify the relation between I and T_{rot}.

(b) The spring constant k can be determined by hanging the provided weights from the end of the spring with the special hook and pointer. Then, you may want to see if the expression for T_{transl} gives the measured period within your experimental uncertainty.

(c) By using the relations given in the references, you may, with some effort, relate T_{transl} and T_{rot} to the physical dimensions, shear modulus, and Young's modulus of the spring.

(d) You might want to identify the normal modes (eigenmodes) of this system under the condition of $T_{rot} = T_{transl}$.

(dh - 1974, jbv - 1990)