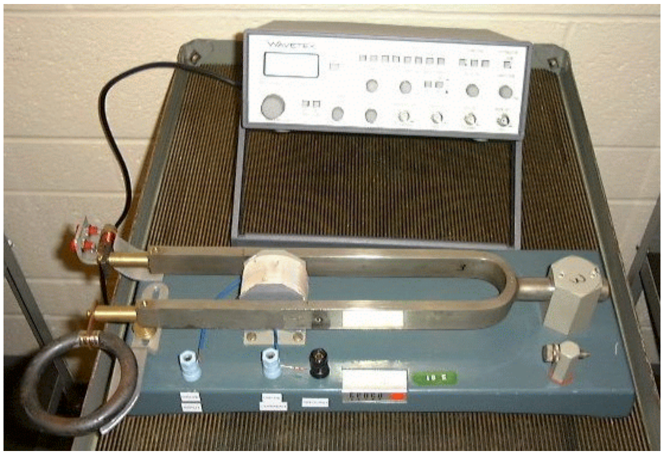
MECHANICAL OSCILLATIONS
RESONANCE AND RINGING IN A TUNING FORK



Objectives:

- to understand how damped resonant systems behave
- to recognize damped oscillations
- to recognize a resonance curve
- to use a number of functions on a digital oscilloscope
- to use an oscilloscope as a tool to measure mechanical phenomena

References

- R. Wolfson & J.M. Pasachoff, *Physics with Modern Physics*, (Addison Wesley 1999); - The chapter on Oscillatory Motion
- R.A. Serway, *Physics for Scientists & Engineers*, (Saunders 1996); - The chapter on Oscillatory Motion
- A.P. French, *Vibrations & Waves*, (Norton 1971); The chapters on Free Vibrations and on Forced Vibrations
- TEKTRONIX web page, *XYZs of Analog and Digital Oscilloscopes*, the URL is www.tek.com/Measurement/App_Notes/XYZs/ (A copy of this booklet is also available in the wicket.)

Introduction

In everyday life, we often see damped oscillatory motion. Examples are the swinging of a pendulum, the bouncing of a car's front-end suspension (particularly visible if the `shocks' are worn out), the vibration of a string of a guitar. Damped mechanical oscillators consist of an inertial mass (m) configured with some sort of spring which supplies a restoring force, that force being proportional to the displacement of the mass (restoring force = k × displacement where k is called the spring constant), and damped by a force proportional to the velocity which is in the direction opposite to the velocity (damping force = γ × velocity where γ is called the damping resistance).

In this experiment you will have an opportunity to play with a rather large tuning fork (not exactly designed to tune your instrument at only roughly C#, about two octaves below middle C). You will find what happens when you hit it, what happens when you keep vibrating it, and what damping does to the way it vibrates. You will also play with an oscilloscope, starting to explore one of the most useful of lab instruments.

Theory

The equation of motion of the mass can be shown to be:

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0 \tag{1}$$

If the mass is driven by a force varying sinusoidally, as $F\cdot\cos\omega t$, then this equation becomes:

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = F\cos\omega t \tag{2}$$

It is often convenient to define two parameters ω_o and Q in terms of m, γ, k. Calling $\omega_o^2 = k/m$ and $Q/\omega_o = m/\gamma$, (note that ω_o has dimensions of radians/second, and Q is dimensionless) then equation (2) becomes:

$$\frac{d^2x}{dt^2} + \frac{\omega_o}{Q}\frac{dx}{dt} + \omega_o^2 x = \frac{F}{m}\cos\omega t \tag{3}$$

Transient Solution:
If there is no sinusoidal driving term ($F = 0$) and if $Q > ½$ (i.e. γ is small enough) then the solution to this equation becomes:

$$x = A\cos(\omega t + \phi)\, e^{-\alpha t} \tag{4}$$

You can check equation (4) is the solution by substituting it in Equation (3) and you will find that $\omega = \omega_o\sqrt{1 - \frac{1}{4Q^2}}$

and $\alpha = \frac{\omega_o}{2Q}$. Q is called the quality factor. For the cases you will be studying in this experiment, $Q >> ½$ and so $\omega \approx \omega_o$. (A and ϕ are constants of integration and depend on the initial conditions.) This solution is called the transient response of the system - it represents what happens when there is no steady driving force but the mass has been disturbed momentarily. Here the mass oscillates with a frequency ω_o, but the size of the oscillations decays exponentially in time.

Steady State Solution:
If the mass is steadily driven sinusoidally by a force $F\cdot\cos\omega t$, then the solution to equation (3) becomes:

$$x = B\cos(\omega t + \theta) \tag{5}$$

with $$B = \frac{F}{k}\, Q\, \frac{\omega_o}{\omega} \frac{1}{\sqrt{1 + Q^2\left(\frac{\omega_o}{\omega} - \frac{\omega}{\omega_o}\right)^2}} \quad \text{and} \quad \tan\theta = \left[Q\left(\frac{\omega_o}{\omega} - \frac{\omega}{\omega_o}\right) \right]^{-1} \qquad \textbf{(6)}$$

You can obtain these conditions by substituting the solution represented by equation (5) into equation (3). This solution indicates that the mass oscillates sinusoidally at the same frequency (ω) as the driving force. Solution (5) and (6) displays resonance. The size of the oscillation depends on the ratio of this frequency to the resonant frequency (ω_o), on the magnitude of the driving force and on the value of Q. The phase of the oscillation of the mass compared to that of the driving force also depends on the ratio of the frequency to the resonant frequency (ω_o) and on the value of Q.

In saying that this steady state solution exhibits resonant behaviour, we mean that, at a certain frequency (ω_o), the system displays maximum response, with little response away from that frequency. The `width' of the resonance, the range of frequencies over which there is sizable motion of the mass, depends on Q.

In this experiment you will be measuring a "pickup" voltage proportional to the velocity of the tuning fork, and thus you will be interested in the time derivative of equation (5):

$$v = \frac{dx}{dt} = C\cos(\omega t + \delta) \qquad \textbf{(5a)}$$

with $$C = \frac{F}{\sqrt{km}}\, Q \frac{1}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}} \quad \text{and} \quad \tan\delta = Q\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) \qquad \textbf{(6a)}$$

Note that all resonant systems, on first being disturbed, exhibit transient behaviour. However, after enough time has passed for the transient behaviour predicted by equation (4) to decay away, the steady state behaviour is all that is seen. This time is of the order of $3/\alpha = 6Q/\omega_o \approx Q/f_o$, where f_o is the resonant frequency in Hz and ω_o is the resonant angular frequency in radians/second ($\omega_o = 2\pi f_o$).

The Apparatus

The mechanical system you will use is a large tuning fork, in which two massive prongs can vibrate towards and away from each other with a restoring force provided by the springy crotch of the fork. These tuning forks have a resonant frequency near 70Hz ($\omega_o \approx 440$ radians/second) and a Q as high as 2,000. This high Q is the result of the good quality steel in the fork, the small amount of air resistance and the massiveness of the fork.

A ferrite magnet is placed on the end of one prong of the tuning fork which moves with the prong in front of a coil of wire, producing a varying magnetic field through the coil. We refer to this coil as the "pickup" coil. You will find, in your Electricity and Magnetism course, that such a changing magnetic field induces a voltage across the coil proportional to the rate of change of that field. You will be using an oscilloscope to measure the voltage on the coil as a function of time, this voltage, then, being proportional to the velocity of the prong of the tuning fork.

Extra damping (in addition to the damping due to air resistance) of the tuning fork is provided by a plate of copper mounted to the other prong of the fork which you can surround by another magnet. You will also find, in your Electricity and Magnetism course, that an electrically conducting plate, moving in a magnetic field will experience a retarding force proportional to the velocity of motion in a direction opposite to that velocity. Thus you can adjust γ in equations (1) and (2), and so Q, by changing the position of the magnet around the plate. γ is made bigger, and thus Q smaller, by moving the magnet closer. This arrangement permits lowering of Q to as low as 40.

For transient behaviour, the tuning fork can be struck by the rubber mallet provided. For a steady state driving force, an alternating current can be passed through another coil (the "drive" coil) located between the prongs of the fork. The force on the prongs is proportional to the current passing through that coil.

The signals depicting the velocity of the prongs (voltage from the pickup coil), and the driving current (voltage across a resistor in series with the drive coil), are viewed, as a function of time, on an oscilloscope. The sinusoidal signal of adjustable frequency for powering the drive coil is provided by a Wavetek function generator. (Note that the frequency shown on the Wavetek signal generator, f, is given in Hz, whereas ω is in radians/second, $\omega = 2\pi f$.)

The Oscilloscope

The oscilloscope is probably the most useful single electronic instrument that we use in the lab. It is a two dimensional voltmeter, and in this experiment you use it to produce a graph on its screen, measuring a couple of voltages as a function of time. Along with an oscilloscope's great usefulness comes the fact that it is a complex instrument. In this experiment we hope you will develop a mastery of a number of oscilloscope functions.

There are very many controls on an oscilloscope and it will take you many times using the instrument before you can take advantage of all of them. Today, while doing your experiment, you will be guided through a number of the main oscilloscope functions. If you are unable to make something happen on the oscilloscope *after consulting the Tektronix reference*, consult your demonstrator.

On all oscilloscopes, the controls can be grouped from the point of view of three basic functions:

- Controls governing vertical (y) motion of the trace on the screen; (vertical position, vertical sensitivity [volts/div], CH1↔CH2 beam selection, DC-AC-Ground input coupling).
- Controls governing horizontal (x) motion of the trace; (horizontal sweep speed [sec/div], horizontal position).
- Controls governing the time base circuits which internally feed the x deflection of the trace; (trigger level, trigger slope, trigger source, trigger mode). The time base is the circuitry that makes the *x* deflection into an actual time axis on the screen and synchronizes that time scale so that the trace's position matches the time of arrival of the signals being measured.

Recognizing these 3 aspects makes for easier understanding of the multiplicity of controls.

The layout of the Tektronix TDS210 involves 7 turnable knobs and a number of buttons, all grouped according to the above systematics. With the knobs you dial-up vertical sensitivities, sweep speeds, vertical and horizontal positions, and trigger levels. Each of the main buttons calls-up a menu on the right side of the screen and the unlabelled buttons beside the menu listed on the screen allow you to select the various functions.

The input to the oscilloscope is via a coaxial cable, consisting of two wires, one being the centre wire and the other being the surrounding shielding wire braid. (Look at the sample of coaxial cable available.) The outer braid is connected to the frame of the oscilloscope and from there to the round pin of the power plug which connects to a water pipe in the basement of the building. It is called the ground wire. The centre wire is connected to the sensitive electronics inside the oscilloscope. The oscilloscope measures the voltage <u>between</u> the centre wire and the outer braid. For convenience, you are provided with leads with a "BNC" connector on the end that goes to the oscilloscope, and, at the other end, with two plugs which are the connections *you* use, the red one connected to the centre wire and the black one connected to the grounded outer braid.

In this experiment you will be using both channel 1 and channel 2 inputs. We do not detail here how to use these, but rather we expect that you will get help from your demonstrator and from the XYZs reference. We do, however, provide suggested oscilloscope settings as a guide for you to get set up. In addition, the following useful functions of the TDS210 should prove helpful:

- You might notice the "RUN/STOP" to freeze the display at any time.

- "AUTOSET" is often useful in case you have pushed too many buttons and have a strange oscilloscope configuration. (Note that "AUTOSET" will not solve all your problems - it merely configures your oscilloscope to match what some programmer thought you might like, and more often than not, that is not what you really want.) It is good practise to push "AUTOSET" before you start your day's measurement as the oscilloscope, when turned on, starts off configured just as it was before it had been turned off. This, then, gives you a fresh start so that you can then set the controls to your own liking.

The "HARDCOPY" button does just that - it gives a printout of what's on the screen. (Please use this button with discretion - decide which is more important, a pile of screen image printouts, or our forests.)

What to Do

1. <u>Transient Behaviour (high *Q*)</u>:
Start with the tuning fork minimally damped by sliding the damping magnet to the position farthest away from the fork. Connect the oscilloscope lead from channel 1 to the pickup coil. Locate the pickup coil as close to its small magnet (on the end of one tuning fork prong) as possible, but not so close that the magnet touches the coil when the fork vibrates. Locate the tuning fork so that the pickup coil is at least 50 cm away from the oscilloscope. (It should also be at least that far from any computer screen.) (If you bring the coil closer, you may find a lot of magnetic `noise' being picked up by the coil, of frequency in the range of 50 kHz.)

To start, use the following oscilloscope settings:

> *CH1 menu and knobs:* Coupling=DC, BW Limit=ON(20MHz), VOLTS/DIV=20.0mV
> *TRIGGER menu and knob:* Source=CH1, Mode=Auto, Coupling=DC, TRIGGER LEVEL=0.00V
> *HORIZONTAL:* SEC/DIV=10mS

Strike the tuning fork horizontally near the end of the prongs with the rubber end of the hammer. Observe the motion of the fork. Observe the sound of the tuning fork. Observe the pattern on the oscilloscope. Do you see evidence of damped oscillations? Does what you see match what you predict from equation (4)?

The following [1(a) → 1(e)] lists five ways you can determine ω_o and α:

> 1(a) You can get a rough measure of ω_o from the pattern on the screen and from the time scale on the oscilloscope. (Recall $\omega_o = 2\pi f_o = 2\pi/T_o$ where T_o is the period of oscillation.) The time scale (sec/div) is indicated at the bottom centre of the screen.
>
> 1(b) You can get a rough measure of the decay constant α and thus calculate Q using a stop watch and timing how long it takes the amplitude of the pattern to decay to $1/e$ if an earlier value. (Note that in equation (4), the amplitude decays to $1/e$ of a previous value when t increases by $1/\alpha$.)

However, the oscilloscope provides you with some more refined tools to help better measure ω_o and α. They can be found in the MEASURE menu and in the CURSOR menu:

> 1(c) For ω_o its best use the MEASURE function and using the sweep speed (HORIZONTAL SEC/DIV) as already suggested.

There are two approaches to finding α:

> 1(d) For α using the stop watch, use a sweep speed around 2.5 ms/div and use the MEASURE function and hand plot a graph of peak-to-peak ("Pk-Pk") voltage as a function of time.
>
> 1(e) For α using only the oscilloscope, use a sweep speed around 2.5 s/div and use the CURSOR function to measure times and voltages. (When using the display at this slow time scale but displaying a higher frequency signal you run into a problem of digital oscilloscopes called *aliasing*. Going into the ACQUIRE menu and selecting the "Peak Detect" option instead of the normal "Sample" option can improve the measurement in this case.)

Try these various oscilloscope tools, asking for help when needed. It is often helpful to freeze the screen with the RUN/STOP button.
For the case 1(e), from the display on the oscilloscope screen of the damped oscillation on the 2.5 s/div scale, measure the peak-to-peak voltage of the pattern as a function of time and plot this data on semi-log graph paper. From the graph, comment on whether you see exponential decay and what value of α you obtain. You could also do the same case 1(d).

2. Steady State Behaviour (high *Q*):

In this section the tuning fork, will be used as it was in the transient case, but now you will drive it with a sinusoidal force. The drive is obtained by connecting the Wavetek Function Generator output to the drive coil on the tuning fork (terminals labelled "*DRIVE INPUT*" and "*GROUND*") and measuring current going through the drive coil on the oscilloscope by connecting oscilloscope CH2 input to the current terminals on the tuning fork (labelled "*DRIVE CURRENT*" and "*GROUND*".

Turn on the CH2 display on the oscilloscope (leaving the CH1 display also on - you should now see two traces). Use the following oscilloscope settings:

CH1 menu and knobs: Coupling=DC, BW Limit=ON(20MHz), VOLTS/DIV=20.0mV
CH2 menu and knobs: Coupling=DC, BW Limit=ON(20MHz), VOLTS/DIV=500mV
TRIGGER menu and knob: Source=CH2, Mode=Normal, Coupling=DC, TRIGGER LEVEL=0.00V
HORIZONTAL: SEC/DIV=10mS

Set the Function Generator frequency to the resonant frequency you observed in part 1, "Transient Behaviour" observations, set the waveshape to sinusoidal, and set the Generator's amplitude knob to full setting. (The Function Generator has three levels of frequency control: the coarsest is with the buttons at the top of the front panel, the intermediate is the knob on the lower left on the front panel, and the finest control is the knob on the back of the instrument.)

Your oscilloscope now displays the velocity of the tuning fork on channel 1, and the driving force on the tuning fork on channel 2.

Scan through a range of frequencies varying by a total of 3.0 Hz, centred on the resonant frequency. Notice the sharp change in amplitude and phase of the tuning fork pickup signal as a function of frequency. Notice how slowly the system responds to changes, such as changes in frequency, or changes in amplitude of the drive. (Stop the motion by grabbing the fork, and then let go and see the slowness of response.) Recall, that we expect that the response time of the tuning fork will be of the order of $3/\alpha \approx Q/f_o$.

Qualitatively draw what the resonance curve looks like, sketching a graph of the pickup coil voltage (proportional to velocity of motion of the tuning fork) as a function of the frequency of the driving signal. Again, the oscilloscope provides a convenient tool to help you in its MEASURE menu. Of particular usefulness is the peak-to-peak value of channel 1 (the pickup coil signal) and the frequency of channel 2 (the driving signal). The Q of the undamped fork is so high that it would require a frequency meter with a precision of one part in 10,000 to carefully scan this curve. In the absence of such a meter, a very rough qualitative sketch is the best you can do. Thus, this time, you have to consider anything derived from this curve as very rough data.

From the resonance curve observation and sketch, estimate $\Delta\omega$, the width of the curve at the half-power points. (The half-power points are defined as the two points on the curve where the amplitude is $\frac{1}{\sqrt{2}}$ of the peak value. $\Delta\omega$ is then the angular frequency difference between these two.) It can be shown from equation (6a) that $\Delta\omega = \omega_o/Q$ so that from your estimated $\Delta\omega$ you can now calculate an estimate of Q.

Do the two values of Q, that you have so far obtained, agree?

3. Transient Behaviour (Low Q):

As mentioned in the introduction, an electrical conductor experiences a mechanically resistive force in a direction opposite to its velocity when it is moved through the poles of a magnet. So, to provide damping so that you can lower Q, we have provided a magnet that can be swung around a copper plate attached to one of the tuning fork's prongs. In order that you may experience this force, take the loose copper plate (that isn't mounted to the apparatus) and pull it or drop it through the poles of the magnet. We hope you are convinced.

Now repeat the observation of transient response as you did in section 1. This time you will want to start with oscilloscope settings similar to the following:

> *CH1 menu and knobs:* Coupling=DC, BW Limit=ON(20MHz), VOLTS/DIV=20.0mV
> *CH2:* turned off
> *TRIGGER menu and knob:* Source=CH1, Mode=Normal, Coupling=DC, TRIGGER LEVEL=20.0mV
> *HORIZONTAL:* SEC/DIV=50mS

These settings will permit you, initially, to see the transient response behaviour. (Note that this time, the trigger mode is different. You might try your earlier setting of the trigger to figure out the difference between "Auto" and "Normal".)

You will probably have to vary some of these settings, particulary the (horizontal) sweep speed, in order to measure frequency and decay constant. What is Q now?

4. Steady State (Low Q):

Now, with this same magnetic damping on the tuning fork, repeat your observations of section 2. This time the resonance curve will be wide enough to measure. The oscilloscope settings will be similar to that section, though the following variation might make observations easier:

> *CH1 menu and knobs:* Coupling=DC, BW Limit=ON(20MHz), VOLTS/DIV=2.0mV
> *CH2 menu and knobs:* Coupling=DC, BW Limit=ON(20MHz), VOLTS/DIV=500mV
> *TRIGGER menu and knob:* Source=CH2, Mode=Normal, Coupling=DC, TRIGGER LEVEL=0.00V
> *HORIZONTAL:* SEC/DIV=5mS

The slightly faster sweep speed makes viewing the oscillations easier, and the increased channel 1 sensitivity is necessitated by the smaller amplitude of vibration of the tuning fork because of the lowered Q.

You will now encounter a noise problem. The increased oscilloscope sensitivity (now 2mV per division) increases the size of the noise picked up by the pickup coil. If you increase the sweep speed of the oscilloscope to about 5μs per division you might see the roughly 56 kHz magnetic signal radiated from the oscilloscope. If you move the apparatus as far from the oscilloscope and adjust its orientation, you may minimize this signal. But then you see more noise, which you might identify by increasing the sweep speed to 25ns per division. Then you might see signals from FM stations (90 MHz) messing up your observations.

The oscilloscope allows you to do tricks to reduce the effect of noise. Going back to the recommended settings for this section, push the ACQUIRE menu button. Try changing from "Sample" to "Average", and try, then, changing from 4 to 16 to 64 to 128 averages. The noise goes down, but because the oscilloscope averages a number of screen samples, the time it takes to form a stable image on the screen increases.

You should be able to decide your optimal way of obtaining your data from the oscilloscope. In the methodology of section 2, but with actually measured data points, graph the resonance curve and find $\Delta\omega$ and thus Q. To do this, scan through a range of frequencies varying by a total of 10 Hz, centred on the resonant frequency. Notice the less sharp change in amplitude and phase of the tuning fork pickup signal as a function of frequency. Notice that the system now responds more quickly to changes, such as changes in frequency, or changes in amplitude of the drive.

Again, we expect that the response time of the tuning fork will be of the order of $3/\alpha \approx Q/f_o$ which is now shorter.

Obtain data and make a graph of the resonance curve on linear graph paper; i.e., plot the velocity of motion of the tuning fork as a function of the frequency of the driving signal. This time the oscilloscope provides a convenient tool to help you in its CURSOR menu. (MEASURE is not as useful with the noise present, but try it anyway.)

From the resonance curve graph, find $\Delta\omega$, the width of the curve at the half-power points. (The half-power points are defined as the two points on the curve where the amplitude is $\frac{1}{\sqrt{2}}$ of the peak value.
$\Delta\omega$ is then the angular frequency difference between these two.) It can be shown from equation (6a) that $\Delta\omega = \omega_o/Q$ so that from $\Delta\omega$ you can now calculate Q.

Do the two values of Q for this damping, that you have so far obtained, agree?

5. <u>Any Other *Q*</u>:

If you wish, you could repeat the above for intermediate values of Q, or you might just play to see what else you might find.

Comments for your Contemplation

Did you find that the tuning fork responded to produce the shapes of the functions predicted by equations (4) to (6)?

Speculate on the experimental limitations presented by the relationship between Q and α to experimenters who want to isolate a very narrow range of frequencies.

Identify other damped mechanical oscillators in every day life. What phenomena provide the damping in these cases?

Can you define in your own words what "resonance" is?

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