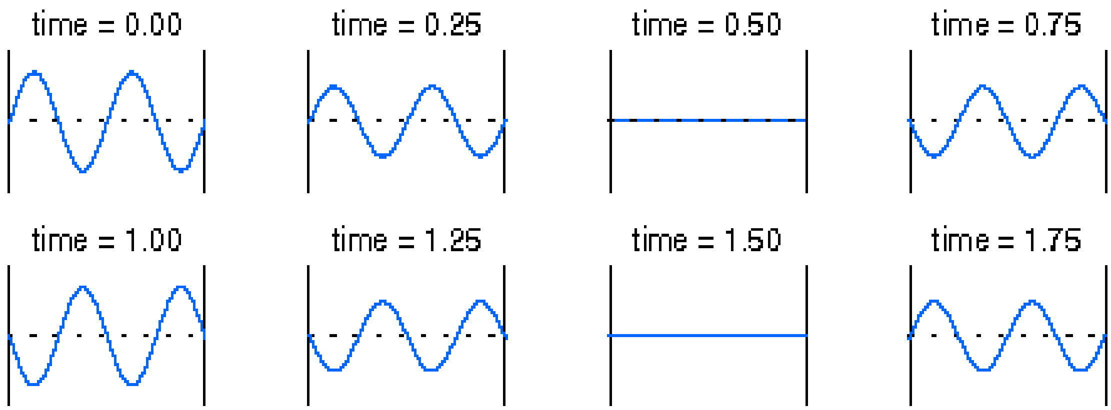
STANDING WAVES & ACOUSTIC RESONANCE

REFERENCES

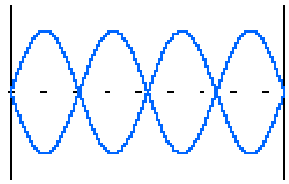
R.H. Randall, *An Introduction to Acoustics*, (Addison-Wesley, 1951), Sect. 7-21, 7-22.
A.B. Wood, *A Textbook of Sound*, (Bell & Sons, 1944), pp.179 *et seq*.
Berkeley Physics Course, *Vol. 3*, *Waves*, §3-2.
Chapter **COMMONLY USED INSTRUMENTS** on *The Oscilloscope* of this Lab Manual

INTRODUCTION

Consider a vibrating string that is fixed on both ends. Because it is fixed on the ends, the only stable vibrations of the string are those with "nodes" at the ends. Such a wave state is called a standing wave. For example, if we took a series of snapshots of a string vibrating in a particular standing wave, it might look like:



In the above at times 0.5 and 1.5, the amplitude of the vibration is zero everywhere. At time 2 the vibration begins to repeat. We *represent* the above vibration with a single figure:



The positions where the string does not move are called nodes. In the above case there are five nodes. The positions where the amplitude of vibration is a maximum are called antinodes.

Here are some other standing wave states for a string fixed at both ends:

The distance from each node to the next, **d**, is related to the wavelength λ of the wave by:

$$\lambda = 2\mathbf{d}$$

The same relationship holds when **d** is the distance from one antinode to the next. It is pretty simple to show that for a string of length **L** the possible wavelengths of a standing wave are:

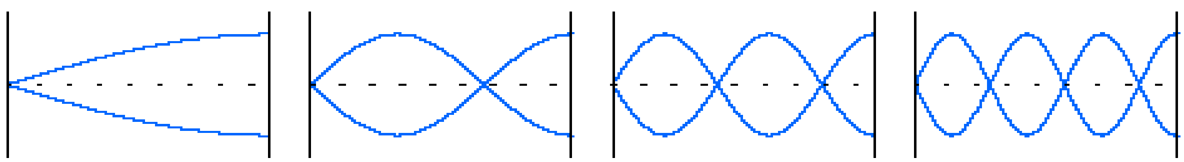
$$\lambda = \frac{2L}{n}, n = 1,2,3,\ldots$$

Standing waves in the basilar membrane of the inner ear are crucial to our sense of hearing. Similar standing waves exist for all kinds of waves, such as light waves and sound waves. For light waves, these standing waves are an important aspect of how a laser works. Standing waves for sound are the topic of this experiment.

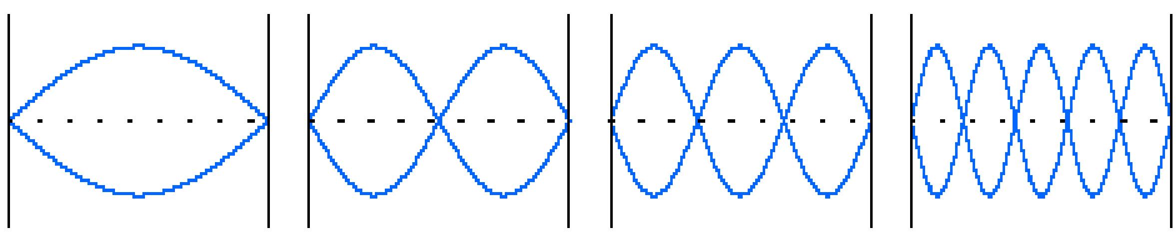
For positions other than the nodes, the string is oscillating with a frequency ν. The relationship between frequency and wavelength for a wave that travels at a speed **v** is:

$$\lambda \upsilon = v$$

If the string is fixed on, say, the left hand side but is free to move on the right hand side, the possible standing waves have a node on the left and an antinode on the right. Here are some examples:



This type of wave can be generated if the "free" end of the string has a small frictionless loop around a vertical fixed wire.



Question: what is the relationship between the wavelength and the length of the string for this type of standing wave?

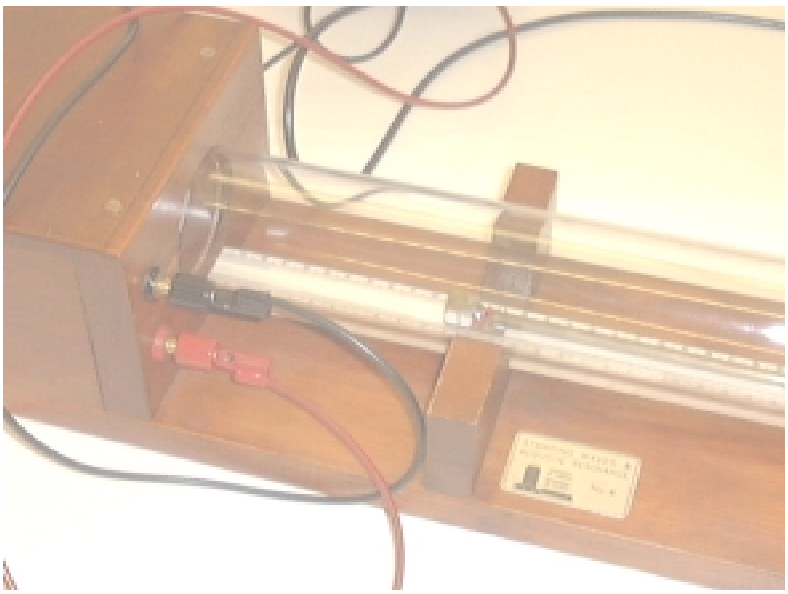
Technical Notes

The reason why these standing waves occur is that when a sine wave traveling, say, to the right strikes the fixed end of the string, it is reflected back to the left. In the course of the reflection, the amplitude is reversed. The "standing wave" is actually the interference pattern between the incident wave and the reflected wave.

For the string that is not fixed on the right hand side, reflection occurs at that side too. However, the amplitude of the reflected wave is not reversed in this case.

APPARATUS

The main part of the apparatus for this experiment is a glass tube with a small loudspeaker at one end. The other end of the tube can be open or closed. A rod inside the tube has a small microphone mounted on the end, so the sound wave inside the tube can be measured at different positions.



A signal generator generates a voltage that varies in time as a sine wave. Both the amplitude and frequency of the wave can be varied. When the signal generator is connected to the loudspeaker, a sound wave is generated whose frequency equals the frequency of the voltage.

An oscilloscope displays input voltages as a function of time. The oscilloscope for this experiment can display two different input voltages simultaneously. You will use one beam to monitor the voltage being delivered to the loudspeaker and the other beam to measure the voltage from the microphone. Information on using the oscilloscope may be found in **Commonly Used Instruments** section of the lab manual; a videotape on the oscilloscope is also available from the Resource Centre in Room 126.

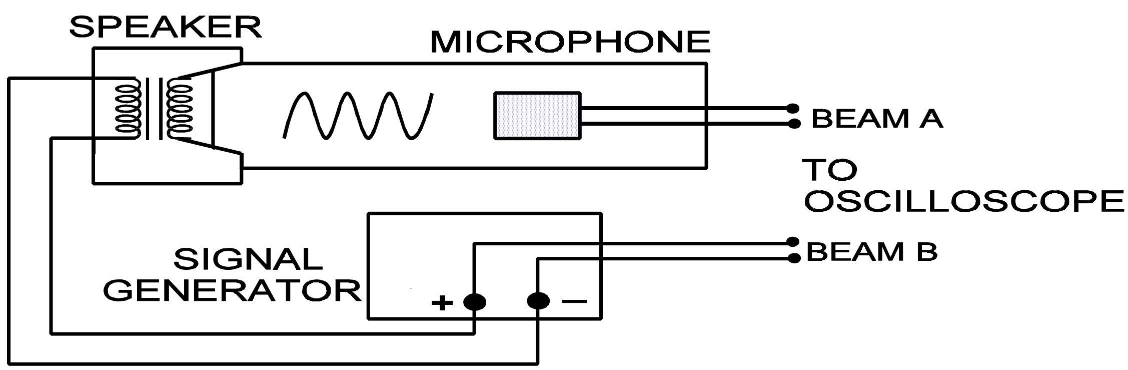
You will wish to know that the microphone measures the *pressure* wave associated with the sound wave. The voltage from the microphone is proportional to the amplitude of this pressure wave. The discussion in the INTRODUCTION section, as applied to sound waves, would be for the *displacement* wave, by which we mean the displacement of the air molecules from their equilibrium positions. If the displacement wave is, say, a sine wave the pressure wave will be its derivative, a cosine wave. Thus when we have a node in the displacement wave the microphone will measure an antinode, and vice versa.

PRELIMINARIES

Connect the signal generator to the oscilloscope. Generate a voltage of a few hundred Hz, and display it on the oscilloscope. In order for the display to be stable, you will probably want to trigger the oscilloscope on the input voltage.
Compare the frequency of the wave as claimed by the signal generator to the frequency as measured with the oscilloscope. Recall that the relationship between the frequency of a wave, in Hz, is related to the period **T** in seconds by:

$$T = \frac{1}{\nu}$$

Connect the output of the signal generator to the loudspeaker. Connect the output of the microphone to the second beam of the oscilloscope and adjust the display so that you can see both the input voltage from the signal generator and the output of the microphone simultaneously. Thus, your apparatus should conform to the following figure:



THE EXPERIMENTS

Part 1

Have the tube closed at both ends. For some frequency between, say, 200 Hz and 2 kHz, adjust the frequency so that a standing wave exists in the tube. One way to adjust the frequency for a good standing wave is to place the microphone as close to the loudspeaker as possible and adjust so that the amplitude as measured by the microphone is a maximum; recall that this corresponds to the displacement of the air molecules from their equilibrium position being a minimum. A secondary adjustment can be made by placing the microphone at the position of a node in the pressure wave and making small adjustments of the frequency to make the measured amplitude as small as possible.

Question: how does the sound level that you hear with your ear close to the tube correlate to whether or not a standing wave exists in the tube? Can you think of why this is so?

When adjusted for a standing wave, we say that there is a resonance between the loudspeaker and the tube.

For the lowest frequencies, the maximum amplitude as measured by the microphone does not occur at the position of the loudspeaker, but a noticeable and measureable amount down the tube away from it. This is your first indication that the simple picture of these waves as described in the **INTRODUCTION** is not quite complete.

From the positions of the nodes and/or antinodes, determine the wavelength of the standing wave. Calculate the speed of sound. Repeat for a few different standing waves. Give a final best value for the speed of sound and compare your result to the accepted value:

$$v_{accepted} = 331 + 0.61t$$

where the speed is in m/s and **t** is the temperature in Celsius.

Is your result within errors of the accepted value?

For one of the higher frequency standing waves, from your measurements of the wavelength determine the length of the tube. Compare to a direct measurement of the length. Are they the same within errors?

Part 2

Open one end of the tube, and adjust the frequency for a standing wave. The standing waves in this case will correspond to the standing waves on a string with one end free to move. From the speed of sound, the frequency of the wave, and your measurements of the positions of the nodes and/or antinodes, calculate the length of the tube and compare to your result of **Part 1**.

As mentioned in the **Technical Notes** above, these standing waves occur because part of the incident sound wave is reflected from the open end of the tube. However, the effective reflection point of the wave is not the exact position of the open end of the tube but is slightly beyond it, and so the effective length of the tube is greater than its real length:

$$L_{effective} = L_{real} + \Delta L$$

where:

$$\Delta L \approx 0.3D$$

where **D** is the diameter of the tube.

Question: does your data confirm the above relationship for the effective length of the tube? A simple answer of "yes" or "no" is not sufficient: you must prove your answer using your data.

Part 3

If the apparatus were perfect, then when the tube is closed on both ends we would not hear any sound outside the tube. Similarly, if the air inside the tube were perfect, all molecule-molecule collisions would be perfectly elastic; this means that as a sound wave travels through the air none of its energy would be converted to heat energy of the air. However, neither the apparatus or the air are perfect.

Say we have a standing wave when the frequency is ν. Then for frequencies close to the "resonant frequency" ν_0 the amplitude of the sound wave is:

$$A(f) = A_o \frac{1}{\sqrt{1 + Q^2\left(\frac{f}{f_o} - \frac{f_o}{f}\right)^2}}$$

where **Q** is called the "quality factor."

Close the tube at both ends and adjust for a standing wave in the range of 200 Hz - 1 kHz. Place the microphone at a maximum in the pressure wave and take data for the amplitude as a function of frequency.

A nearly trivial amount of algebra shows that the amplitude is $1/\sqrt{2}$ times the maximum amplitude when the frequency is:

$$\nu = \frac{\nu_0}{2Q}(1 \pm \sqrt{1 + 4Q^2})$$

or:

$$Q = \frac{\nu\nu_0}{\nu^2 - \nu_0^2}$$

From your data, then, what is the quality factor? You will want to know that the error in **Q** is given by:

$$\Delta Q^2 = \left(\frac{\nu_0}{\nu^2 - \nu_0^2} - \frac{2\nu^2\nu_0}{(\nu^2 - \nu_0^2)^2}\right)^2 \Delta\nu^2 + \left(\frac{\nu}{\nu^2 - \nu_0^2} + \frac{2\nu_0^2\nu}{(\nu^2 - \nu_0^2)^2}\right)^2 \Delta\nu_0^2$$

You will probably also be pleased to know that this is the most complicated error calculation you are probably going to confront in this laboratory.

In at least a qualitative way, show that your data is consistent with the above equation for A(**ν**). Can you think of how you might improve your demonstration of consistency?

AUTHOR

This Guide Sheet was written by David Harrison, October 1999.

Preparatory Questions.

Note: We hope that the following questions will guide you in your preparation for the experiment you are about to perform. They are not meant to be particularly testing, nor do they contain any "tricks". Once you have answered them, you should be in a good position to embark on the experiment.

1. Suppose the tube is open at one end, closed at the other. Starting at the lowest frequency (i.e. the longest wavelength), draw plots of the first four standing wave patterns you would expect to see.

   (*In the diagram opposite, the lowest frequency standing wave has been indicated for you. Complete the diagram, indicating the relationship between the length of the tube and the wavelength for each frequency.)*

2. The equation for the measured amplitude as a function of frequency (i.e. *A(f)*) is given in Equation (1). At what value of *f* is this amplitude a maximum; what is its value then?
3. Prove Equation (2) in the guide sheet for this experiment.
4. Rearrange Equation (1) in the guide sheet into the form $y = mx + b$. What corresponds to *m* and what to *b* ?
5. The microphone you will be using indicates a *maximum* signal when the amplitude of the standing wave in the tube is a *minimum* (i.e. an amplitude node). Why is this so? (*Hint: read what the guide sheet says about the microphone).*