### THE SPEED OF LIGHT

### REFERENCES

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# **INTRODUCTION**

This experiment is a modification of Foucault's method of 1862 in which there is no requirement for the use of a concave mirror. Foucault achieved a measurement of the speed of light, c, to an accuracy of  $\pm 0.17\%$  using a baseline of only 20 m. Although the technique you use is similar, you can probably expect to achieve accuracies of  $\pm 5\%$  in one afternoon's work. You will find, as did Foucault, that there is a limit to the accuracy achievable in a technique dependent on the measurement of the displacement of a light beam. It is interesting, however, considering the magnitude of the velocity being measured, that one can quite easily measure c in one afternoon.

The other challenge of this experiment is the solving of the problem of how to set-up an apparatus systematically. If you work out your system of alignment carefully, you can perform the experiment easily. However, if you do not proceed systematically, you could spend days without achieving one observation.

So have fun!

#### THEORY

In principle, the apparatus is that of Figure 1. (Our actual set-up achieves a longer flight path for the light by inserting an extra mirror between Mo and L so that the light beam can double back in the room.)





Light from a source S falls onto the rotating mirror  $M_r$  and is focussed by the lens L to form an image I on the surface of the mirror  $M_o$ . The light from this image, reflected by  $M_o$ , will form a second image exactly coincident with the original object S, since I and S are conjugate points of the lens L. Note that this is true for all rays that pass through L and reach  $M_o$ , even though these rays will not generally retrace their paths exactly (as they did in the original Foucault method), because of the focussing properties of L.

It is important to note that the position of the final image on return to the source location S is independent of the angle  $\beta$  of rotating mirror  $\mathbf{M}_r$ , although the position of the image I on the face of mirror  $\mathbf{M}_o$  will vary with angle  $\beta$ . (Again, note that this is true only if the rays do pass through lens L and do strike the surface of  $\mathbf{M}_o$  - otherwise the light just won't make it back to the location of S.)

The above considerations are true for  $M_r$  stationary at any reasonable angle  $\beta$  or rotating at

some low value of angular velocity  $\omega = \frac{d\beta}{dt}$ . However, if  $\omega$  is sufficiently large so that the

mirror  $\mathbf{M}_{r}$  has rotated through and angle  $\delta\beta$  in the time the light takes to travel from  $\mathbf{M}_{r}$  to  $\mathbf{M}_{o}$  and back, the final image will be displaced sideways from the original source position S.

Referring to Figure 2, if the rotating mirror is initially at position 1 the slit produces an image P in  $\mathbf{M}_{\mathbf{r}}$ . The light, after reflection from  $\mathbf{M}_{o}$  forms another image at the same position P on its return. But if  $\mathbf{M}_{\mathbf{r}}$  is now at position 2, the light is reflected at a larger angle to form an image at S. If the mirror has rotated through an angle  $\delta\beta$ , the angle of change of direction of the final image as viewed from the position of  $\mathbf{M}_{\mathbf{r}}$ ,  $\angle \mathbf{S}\mathbf{M}_{\mathbf{r}}\mathbf{S}$ , is 2 x  $\delta\beta$ . Thus, the displacement x of the final image, due to change in mirror angle from when the light leaves  $\mathbf{M}_{\mathbf{r}}$  to when it returns is just  $x = 2\ell \times \delta\beta$ , assuming  $\delta\beta$  to be small, and where  $\ell$  is the distance from  $\mathbf{M}_{\mathbf{r}}$  to S.



Figure 2.

If the optical path length between  $\mathbf{M}_{r}$  and  $\mathbf{M}_{o}$  is d, then the light, travelling at a speed c, takes a time  $\delta t = \frac{2d}{c}$ 

between reflections at  $M_r$ , so that if the mirror is rotating at an angular velocity  $\omega$ , then

$$\delta\beta = \omega \times \delta t = 2\omega \frac{d}{c}$$
 and thus  $x = \frac{4\ell d}{c}\omega$  (1)

The determination of c thus reduces to the measurement of two fixed distances, a variable angular velocity, and a dependent displacement of an imaged light beam.

#### THE EXPERIMENT

The apparatus should be arranged in a manner shown below.



Notice that we have folded the path between  $M_r$  and  $M_o$  by inserting mirror  $M_i$  at the end of the other lab room, in order to extend the path length d and thus effectively increase x.

The rotating mirror assembly has a photocell attachment which can be connected to an oscilloscope to measure the time of rotation of the mirror to deduce  $\omega$ . In determining  $\omega$  be sure to check whether the rotating mirror is single or double sided as this will affect your calculations. You might wish to calibrate the oscilloscope time base, either against a frequency counter or against the 60 Hz power line frequency.

Although the method requires merely a small intense light source (such as a small intense focused light and a slit), we have provided you with a laser light source. Lasers have the advantage of lots of intensity, all directed in the direction you want. However they have the disadvantage of having too small an angular spread for this experiment so that if the rotating mirror gets even slightly misaligned (as can happen at high rotational speeds if the motor shaft is at all out of balance) then the beam can end up completely missing mirror  $\mathbf{M}_{o}$ .

Note that the mirrors used in this experiment are aluminized-front-surface mirrors. They do not have a protective overcoating. Great care should be taken not to touch the mirrored surface nor should any attempt be made to clean them, otherwise damage to the mirror will surely result.

# CAUTION: If viewed directly, the beam from even a low power laser can cause permanent damage to your vision. Never look directly into the beam without a diffusing screen between you and the laser.

# STRATEGIES

As mentioned in the introduction, most of the difficulties in this experiment lie in the process of arriving at a good optical arrangement. The following steps may be used as a guide.

- 1. Place the rotating mirror  $\mathbf{M}_{r}$  and the fixed mirrors  $\mathbf{M}_{o}$ ,  $\mathbf{M}_{i}$  to get the maximum path length possible given the finite size of the space. This implies placing  $\mathbf{M}_{r}$  and  $\mathbf{M}_{o}$  close to one wall, and  $\mathbf{M}_{i}$  close to the facing far wall in the other room. Place the laser source,  $\mathbf{S}$ , on its stand at about 2.5 m from the rotating mirror.
- 2. Measure the total length of the light path from laser source S to the farthest mirror  $M_0$ .
- 3. Calculate the position for lens L to make S and  $M_o$  optically conjugate (i.e., L causes an object at S to form an image at  $M_o$ ). [You may notice that the lens has a value for the focal length of 4750 mm written on it. My measurements, using autocollimation, gave a value of  $(4.80 \pm 0.03)$ m.]
- 4. With lens L *removed* from the light path, adjust the angles and positions of S,  $M_r$ ,  $M_i$  and  $M_o$  so that light from the laser traces the full path from S to  $M_o$ , hitting the centres of each mirror in turn, and returns via the same path. The following comments may help you with this line-up process:

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- Note that the rotating mirror  $\mathbf{M}_{\mathbf{r}}$  can be rotated by hand using the removable knob at the top of the motor assembly. *Caution*: it is important that the knob always be removed before the motor is turned on. Running the motor with the knob inserted may bend the motor shaft.
- You may do fine adjustment of mirrors by gently rotating their base stands and by changing their tilt by using their base stand adjusting screws.
- In directing the beam, it is often easier to rotate the appropriate mirror rather than run long distances to translate another mirror sideways or vertically.
- Even one person working alone on the experiment should be able to line-up the beam with at most two walking trips up and down the full light path.
- It is useful to start your line-up by adjusting all mirrors and the laser source to be the same height.
- 5. Place L in its calculated location and adjust its vertical and transverse position so that the beam again traverses the path

$$S \Rightarrow M_r \Rightarrow L \Rightarrow M_i \Rightarrow M_o \Rightarrow M_i \Rightarrow L \Rightarrow M_r \Rightarrow S$$

- 6. By this point in the set-up, you have the beam returning to the laser source S. If you were to now run the motor to rotate  $\mathbf{M}_{r}$  you would not be able to make measurements since this would require that your eye be inside the laser. Thus you must now insert the "scoop-off" small mirror  $\mathbf{M}_{s}$  between the laser S and  $\mathbf{M}_{r}$ , angled in such a way that the returning light beam gets reflected on to your observing scale rather than returning to the laser S. The "scoop-off" is achieved by first doing a very slight adjustment of laser S so that its beam strikes  $\mathbf{M}_{r}$  near its top and then doing a very slight adjustment of the tilt of  $\mathbf{M}_{r}$  and the tilt of  $\mathbf{M}_{o}$  so that the teturning beam strikes  $\mathbf{M}_{r}$  near its bottom.
- 7. Now  $\mathbf{M}_{s}$  may be placed near  $\mathbf{M}_{r}$  such that it intercepts only the beam travelling from  $\mathbf{M}_{r}$  headed towards  $\mathbf{S}$ , but does not intercept any of the three other light beams arriving at or leaving  $\mathbf{M}_{r}$ . Adjust the angle of  $\mathbf{M}_{s}$  so that the outgoing beam hits the viewing scale. position the viewing scale at such a distance that the light spot is small in size and does not move from side to side when  $\mathbf{M}_{r}$  is rotated through small angles. Notice that theory says that this position should be achieved when the light path distance from  $\mathbf{M}_{r}$  to  $\mathbf{M}_{s}$  to the viewing scale is the same as the distance from  $\mathbf{S}$  to  $\mathbf{M}_{r}$ .
- 8. You could run your experiment using the viewing scale. However, for greater precision you may substitute the scale and magnifying eyepiece assembly for the viewing scale. Also you may insert a slit directly in from of the laser. Choose the slit width to optimize clarity, sharpness and intensity of image as seen in the viewing eyepiece.

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#### **DATA INTERPRETATION**

The form of equation (1) immediately suggests a graphical way of interpreting your data. Note that x in equation (1) represents the displacement of the final light spot from its position when the mirror is not rotating. Your measuring scale will probably be arbitrarily positioned with its zero not corresponding to the position of the spot for a stationary  $M_r$ . Thus this equation is better written as:

$$x = \frac{4\ell d}{c}\omega + x_o$$

Do not try to determine  $x_0$  with the mirror stationary as the brightness of the spot is so much greater than when the mirror is rotating that the data taken under the two intensity conditions is not comparable.

This experiment is ideally suited to plotting your data while you take it. In so doing make sure that you have sufficient points to obtain a useful result. Take some of your data with  $\omega$  increasing, and some with  $\omega$  decreasing and keep track, on your graph, of each set of data. Be sure to take points for as large a range of values of  $\omega$  as is practical.

(jv - 1988)