

OSCILLATIONS OF A SPHERE ON A CONCAVE SURFACE

INTRODUCTION



On first glance the motions of a sphere on a concave spherical surface of radius of curvature R might appear to be that of a simple pendulum of length R . However, a quick recognition of the effects of the moment of inertia of the sphere leads to a different conclusion. If a sphere of radius a undergoes small oscillations on a surface of radius R , without slipping, the period is given by:

$$T = 2\pi \left[\frac{7}{5} \left(\frac{R - a}{g} \right) \right]^{\frac{1}{2}}$$

(You must derive this relation as part of your lab write-up.)

This experiment enables a simple precise measurement of the radius R .

THE EXPERIMENT

The apparatus will allow a measurement of R . This can be best done by using several different spheres on the surface and plotting T^2 versus a .

The above simple measurement assumes that the sphere moves in a vertical plane, passing through the lowest point on the spherical surface. You should consider (and, if you wish) investigate what happens if the sphere executes some other trajectory on the spherical surface.

The measurement of R can be checked by using the spherometer, which is a purely calliper device. Our spherometers have rather limited accuracy - try to evaluate the accuracy and precision of your measurements.

In order that you can further check your measurements of R , we have had the radius of curvature of the spherical surface measured using high precision equipment. The value of the radius of *one* surface is indicated on the side of the glass disk.

USING THE SPHEROMETER

Find the *zero* reading by taking readings when the four feet are all in contact with the large plane glass surface. Delicate touch is required for an accurate reading. If the central foot is too low the instrument readily rotates around it. If it is too high the other feet squeak when the spherometer is moved along the surface.

Now take readings for the spherical surface to be measured, again making certain that all four feet contact the surface. Get an accurate reading by oscillating between positions when the central foot is too low and too high, gradually cutting down the size of the up and down shift of the central leg. The difference between the zero reading on the plane glass and the present reading gives us the distance between the foot of the central leg and the plane passing through the feet of the outer three legs.

Referring to the figure, if *A* is the central foot, *D*, *E*, *F* the outside feet, *B* the centre of the triangle $\triangle DEF$, *C* the other end of the diameter through *A* and *O* the centre of the sphere and defining:

R = radius of the spherical surface = distance OA = distance OD

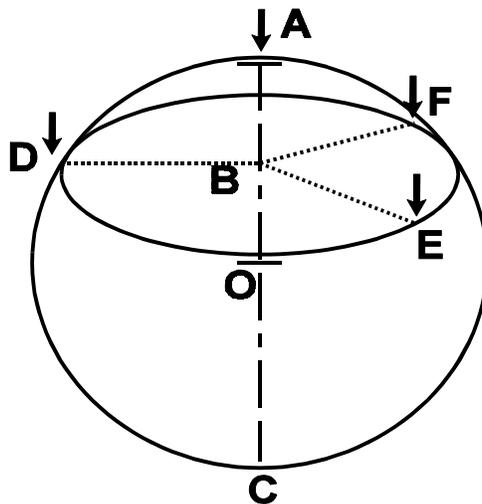
h = distance AB (measured by spherometer)

d = distance DB = distance EB = distance FB

you can show that:

$$R = \frac{d^2}{2h} + \frac{h}{2}$$

(derive this equation as part of your lab write-up)



Measure d with vernier calipers. To do this bring the central foot down to the position on the level with the others, press the spherometer lightly on the page of your notebook and measure the distance between the pricks. Record the three distances and take the mean.

(gmg - 1971, jv - 1988, 1990)