THE MILLIKAN OIL-DROP EXPERIMENT

REFERENCES

1.R.A. Millikan, The Electron (photocopied excerpts available at the Resource Centre).

2.Instruction Manuals for Leybold-Heraeus apparatus (available at the Resource Centre).

(2 weights)

INTRODUCTION

This experiment is one of the most fundamental of the experiments in the undergraduate laboratory. The experimental apparatus is patterned after the original apparatus, made and used by R.A. Millikan to show that electric charge exists as integral multiples of "e" the charge on a single electron. Historically, this experiment ranks as one of the greatest experiments of modern physics.

GETTING STARTED

This method first described in 1913, is based on the fact that different forces act on an electrically charged oil drop moving in the homogeneous electric field of a plate capacitor. When the plate capacitor's electric field intensity is E, the following forces act on a droplet of charge Q:

- gravitational force $m_{oil}g$, where m_{oil} is the mass of the oil drop,

- buoyant force $m_{air} g$, where m_{air} is the mass of air displaced by the oil drop,

- electric force QE,

- and, only if the droplet, considered in this case as a sphere, moves against the ambient air: Stokes' resistance force. Stokes' Law states that for a spherical object of radius r moving through a fluid of viscosity η at a speed v under laminar flow conditions, the viscous force F on the object is given by $F \approx 6\pi r\eta v$. The viscous force always opposes the motion and is, of course, responsible for the steady terminal velocities observed when a drop falls in air.

We suggest the following method to find the charge on an oil drop:

(i) measure the fall velocity v_1 of a droplet in the free space (at zero voltage) and also

(ii)   the rise velocity v_2 of a droplet at a definite voltage.

Write down the equation for a charged oil drop falling freely (in zero electric field) in air under gravity and the equation for the same drop moving in the opposite direction due to an applied electric field. Eliminate the radius of the drop from these two equations to obtain an expression for the charge on the drop in terms of measurable quantities and the constants given below. You should obtain:

$$Q = (v_1 + v_2)\frac{\sqrt{v_1}}{U}\eta^{3/2}\frac{18\pi d}{\sqrt{2g\rho}}$$ (1)

where $\rho = \rho_o - \rho_a$. (See APPENDIX for the details of the derivation.)

The following values are substituted for η, ρ, d and g in your experiment:

the density of the oil $\rho_o = 875.3$ kg/m^3
the density of air $\rho_a = 1.29$ kg/m^3
the acceleration due to gravity $g = 9.80$ m/s^2
the viscosity of air at room temperature and 1 atm $\eta = 1.81 \times 10^{-5}$ N·s/m^2
the separation of the parallel plates $d = 6.0$ mm.

The following final equation results:

$$Q = (v_1 + v_2)\cdot\frac{\sqrt{v_1}}{U}\cdot 2\cdot 10^{-10}$$ (A·s).   (2)

It is also possible to measure the charge on the oil drop by a method that adjusts the electric field till the drop is held stationary. Although this is a conceptually simpler method, it is experimentally more difficult. If you are interested, we have included the relevant details of this in the Appendixes.

The smallest division on the scale covers about 1/20 mm in real space. The telescope eyepiece should be adjusted to bring the scale into sharp focus. Play with the equipment and become familiar with it before you take any serious measurements.

NOTES

The main object of the experiment is to demonstrate the quantization of charge. Since some of the values of the physical constants above are approximate you should not worry too much if the value of *e* you obtain is outside the error range you predict from your measurement errors, (you will note the absence of quoted errors in these quantities!). You may neglect the buoyancy of air.

You will need to take measurements on about 75 drops to demonstrate the quantization of electric charge; plot the results on a histogram.

We suggest that you select drops with more or less the same radius (same terminal velocity). If in doubt, ask your demonstrator.

Before starting a longer series of measurements, it is best to choose droplets that have small values of charge. How would you decide which these are?

Avoid using drops of very small radius. Why? Estimate the radius of typical droplets in your experiment (using equation 2A in Appendix).

Please note that the droplet falling in the field-free space rises in the microscopic image and the droplet rising in the presence of the electric field falls in the microscopic image.

Perhaps the easiest way to do the experiment is to measure, for each drop, (i) the terminal velocity v_1 of each drop at zero voltage, (ii) and the rise velocity v_2 of a droplet at a definite voltage (method 1).

After determining the velocity of the droplets, calculate the charge Q. Represent the results in form of a histogram (number of measurements within a range of 10^{-20} A·s versus $Q/(10^{-20}$ A·s)) and extract a value of the electronic charge. The elementary electronic charge *e* is obtained by forming the largest common divisor from the different charge values.

You can also use the alternate method mentioned earlier (method 2). In this case set voltage so that the chosen droplet is held stationary. Try to suspend the droplet in the lower part of the observation field. Read off the voltage. Simultaneously switch of the voltage and start the stop-clock to measure the terminal velocity of the droplet falling in the field–free space. We suggest that you select drops with similar charge.

EXPERIMENTAL MAGNIFICATION OF THE MICROSCOPE OBJECTIVE

When an oil drop moves along a distance x of micrometer scale division ($=$x 10^{-4} m), the actual distance traveled s, taking into account the objective magnification m:

is $s = \frac{x}{m}10^{-4}$ m.

Our instrument has 2.2-fold magnification. Using the transparent rule with mm divisions you can determine the microscope magnification yourself. Remove the Millikan chamber with its acrylic glass cover from the top of the stand rod. Place the rule with mm graduation vertically against the now visible centering rod. Adjust the microscope by means of knurled screw, so that the mm graduation of the transparent rule is sharply visible. The magnification can be calculated by comparing the micrometer scale in the eyepiece (0.1 mm between the divisions) with the mm graduation of the transparent rule. Replace the Millikan chamber with the acrylic glass cover again in position.

QUESTIONS TO CONSIDER

Is the motion of the very smallest of the oil drops at all unusual? Have you any explanation?

Give some indication as to the limits of your measurements when compared to those of Millikan. Would you expect your measurements to be good as his?

(gmg-1990,jbv-1994,tk-1996.1998, ta-2000)

APPENDIXES

I. Motion of charged oil drops in air

(a) For a droplet falling through a field-free space with the terminal velocity v_1 the following rule of forces applies (Figure 1):

$$V(\rho_o-\rho_a)g-6\pi r v_1 \eta \quad = \quad 0$$

or

$$(4/3)\pi r^3 (\rho_o-\rho_a)g-6\pi r v_1 \eta = 0 \qquad\qquad 1A)$$

resulting in $r = \sqrt{\dfrac{9 v_1 \eta}{2 g \rho}}$ $\qquad\qquad$ (2A)

(where $\rho = \rho_o - \rho_a$).

$$F=6\pi\rho\eta v_1$$

The forces on a droplet falling through a field-free Space with the terminal velocity v_1

$$F_b=m_{air}\,g$$

$$v_1$$

$$F_g=m_{oil}\,g$$

Figure 1

(b) With U= voltage between the plates of the Millikan chamber, d=plate spacing, for a droplet floating in the chamber under the influence of an electric field of field strength $E=U/d$ the following relation applies:

$$V(\rho_o-\rho_a)g-QE=0$$

or

$$(4/3)\pi r^3(\rho_o-\rho_a)g-QU/d=0 \qquad\qquad (3A)$$

(c) For a droplet moving upward with the terminal velocity v_2 under the influence of an electric field of field strength E (Figure 2):

$$V(\rho_o-\rho_a)g-QE+6\pi r v_2\eta=0$$

or

$$(4/3)\pi r^3(\rho_o-\rho_a)g-QU/d+6\pi r v_2\eta=0 \qquad\qquad (4A)$$

$F_e=QE$

$F_b=m_{air}\,g$

v_2

$F=6\pi\rho\eta v_1$

$F_g=m_{oil}\,g$
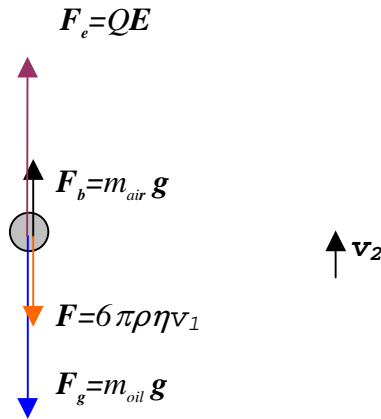
Figure 2

The forces on a droplet moving upward with the terminal velocity v_2
under the influence of an electric field E

The method we suggest to use (method I):

Combining the equations (1A) and (4A) and solving for Q gives:

$$Q=(v_1+v_2)\frac{\sqrt{v_1}}{U}\eta^{3/2}\frac{18\pi d}{\sqrt{2g\rho}}$$

The alternate method (method 2):
Combining the equations (1A) and (3A) and solving for Q gives:

$$Q=\eta\frac{6\pi d v_1}{U}\sqrt{\frac{9\eta v_1}{2\rho g}}$$

APPENDIX II. An alternate method to find Q

 is to measure for each drop
(i) the terminal velocity v_1 of each drop at zero voltage
and
(ii) the voltage U required to just stop the same drop from falling.

Write down the equation for an oil drop falling in air under gravity and that for a charged oil-

drop whose weight is exactly balanced by an applied electric field; eliminate the radius of the drop from these two equations to obtain an expression for the charge on the drop in terms of measurable quantities and the constants given below. You should obtain:

$$Q = \eta \frac{6\pi d v_1}{U} \sqrt{\frac{9\eta v_1}{2\rho g}} \qquad (3)$$

When substituting the values *for* η, ρ, d and g indicated already, one obtains the following final equation which enables relatively quick calculation of the charge values:

$$Q = 2 \cdot 10^{-10} \frac{v_1^{3/2}}{U} \quad (\text{A·s}) \qquad (4)$$

III. Leybold-Heraeus apparatus

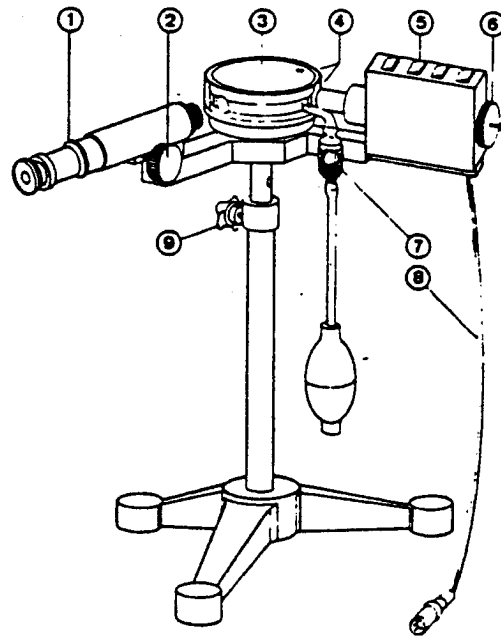Fig. 1

(1) Measuring microscope with micrometer eyepiece
(2) Knurled knob for microscope adjustment
(3) Millikan chamber (plate capacitor) with acrylic glass cover
(4) Socket pair to connect the d. c. voltage for the plate capacitor
(can be tapped from socket pair ⑪, adjustable via knob ⑭)
(5) Illumination device
(6) Knurled knob for lamp adjustment
(7) Oil atomizer with rubber ball in resilient holder (one bottle with oil included in scope of delivery)
(8) Connecting cable for lamp voltage (from multiple socket ⑤)
(9) Screw for height adjustment (to adapt the microscope to the eye level of the experimenter)