Free Fall

PREPARATORY QUESTIONS

1. You measure the initial (s_1) and the final (s_2) positions of a falling object with a tape measure, which has a millimeter scale. The distance s covered by the object is the difference between these two values. What are the reading errors in s_1 and s_2?

2. Now, take into account additional error in the metal scale that is, according to the manufacturer, one part in 4000. Give an expression for the total error in s.

3. The reading error of the timer is 0.05 ms. What is the error in a calculated value of average velocity if the measurements of positions and time were done for just one run?

4. If air resistance is not negligible, you can use the equation below for the distance vs. time and the coefficient of air resistance α:

$$s = s_0 + v_0 t + \frac{(g - \alpha v_0^2) t^2}{2} - \frac{\alpha v_0 (g - \alpha v_0^2) t^3}{3}$$

If you fit distance versus time to a third-order polynomial,

$$s = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

the fitter will give values and errors for the coefficients a_1, a_2 and a_3. Write the equations for v_0, g and α in terms of a_1, a_2 and a_3; and the equations for the error in g and α in terms of errors in a_1, a_2 and a_3.

INTRODUCTION

The "modern" study of objects in free fall near the Earth's surface was begun by Galileo some 400 years ago. In this experiment, you will use the free fall of an object to determine the acceleration due to gravity g. Another goal of the experiment is to study the effect of air resistance.

When an object is in free fall near the Earth's surface and air resistance is considered negligible we can relate the distance s the object falls in a time t to its initial position s_0 at time $t = 0$, the initial speed v_0 at time $t = 0$, and the acceleration due to gravity g according to:

$$s = s_0 + v_0 t + \frac{g t^2}{2} \qquad (1)$$

If you fit distance s versus time t to a second order polynomial ("powers" of 0, 1, 2 in the language of the fitter), the fit is to:

$$s = a_0 + a_1 t + a_2 t^2 \qquad (2)$$

You can easily figure out the relationship among constants of equations (1) and (2).

The apparatus for this experiment is sufficiently precise to display that for some objects air resistance is not negligible. The air resistance exerts an upward force F, which causes an upward acceleration $a_{air\ resistance}$ on the object of mass m:

$$\frac{F}{m} = a_{air\ resistance} \qquad (3)$$

In fluid dynamics this force is called *drag* [1: pp.150-156; 2: pp.167-171]. Drag is explained by *viscosity* of the medium surrounding the moving object. Drag has two components: 1) skin-friction drag due to the force of friction between the surface of the object and molecules of the medium; and 2) pressure drag, which appears when eddies and whirlpools are formed behind the moving object that produce the decrease of pressure behind the object compared to the head-on pressure. Fig.1 [3] shows objects of different shape and relative size moving from the right to the left in fluid with viscosity. The shape of the object influences the relative contribution of skin-friction drag and pressure drag in the total drag force.

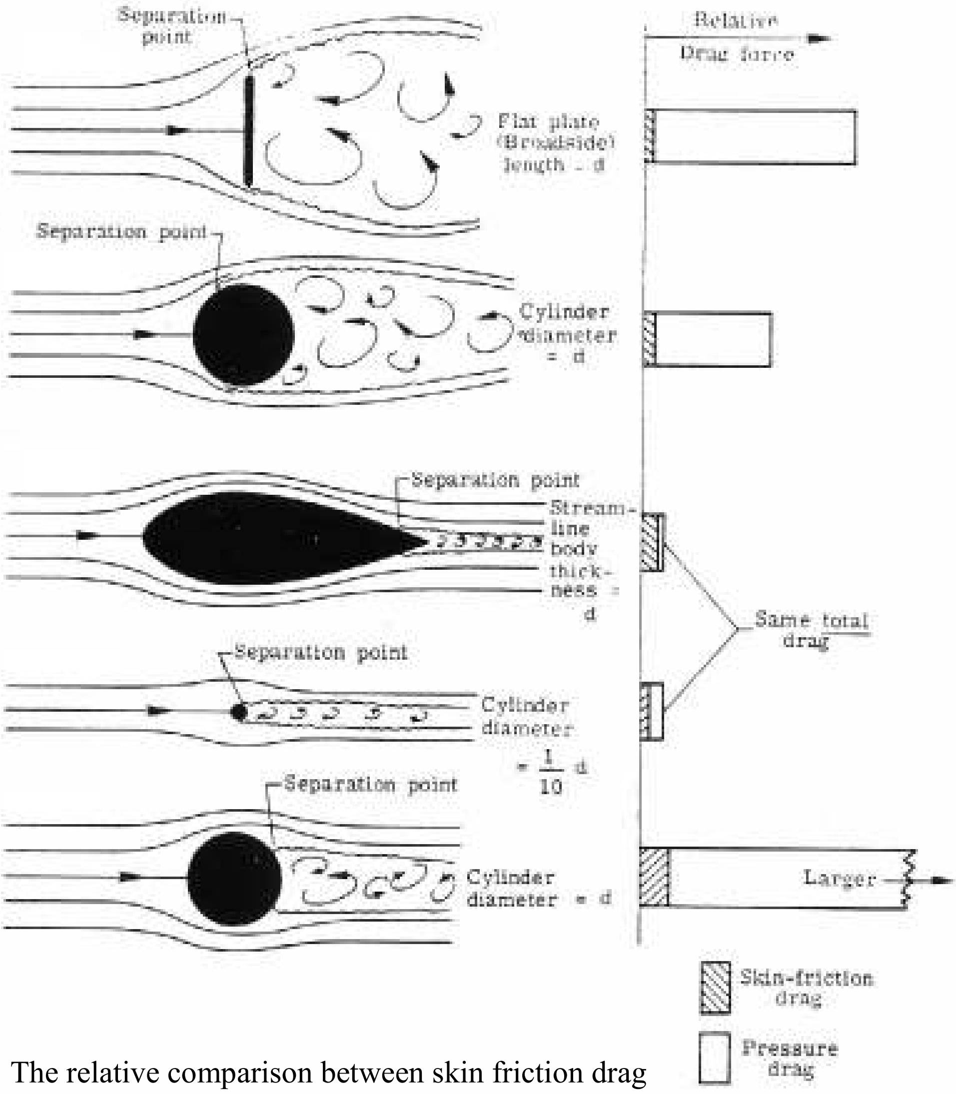


Fig.1. The relative comparison between skin friction drag and pressure drag for various aerodynamic shapes [3].

Note that the drag of the flat plate and the circular cylinders is dominated by pressure drag, whereas, in contrast, most of the drag of the streamlined body is due to skin friction.

Skin friction is directly proportional to the relative velocity of the object and the medium, and plays the main role in resistance with respect to an object moving slowly in medium with great viscosity. Pressure drag is proportional to the square of velocity and is more significant for the same object moving with greater velocity in less viscose media. The air can be assumed to have low viscosity.

Thus, the upward acceleration due to the air resistance depends on the speed, size and mass of the object and on its surface roughness and shape. In the experiment with freely falling object, the upward acceleration is *approximately* equal to:

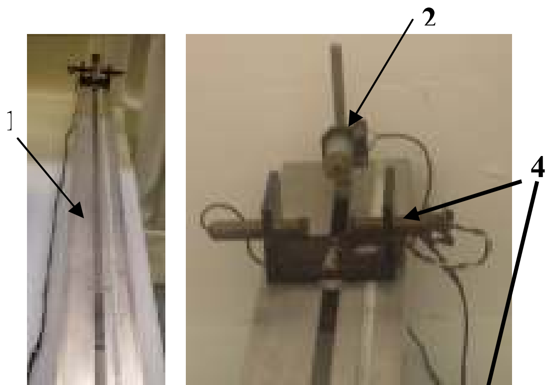
$$a_{air\ resistance} = -\alpha v^2 \qquad (4)$$

Instantaneous acceleration is no more constant and the simple relation between distance and time becomes more complex.

You can try to write the equation of motion basing on the second Newton's law and find the differential equation to be solved to obtain the expression for distance traveled by the falling object as a function of time. This differential equation can be solved analytically and the position of the falling object can be obtained as a function of time t; position s_0 and velocity v_0 at the time instant t = 0; the acceleration due to gravity g; and coefficient α of the equation (4). Values g and α can be combined in $v_{term} = \sqrt{g/\alpha}$, the terminal speed [1: p.152; 2: p.170] of the falling object. This value is the maximum speed that can be theoretically achieved by the object when the drag force becomes equal to the force of gravity.

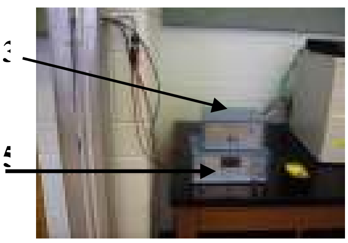
Because: 1- α cannot be measured directly, 2 - equation (4) is accepted but not approved, and 3 - all the above mentioned parameters are measured with errors, it would be easier and more accurate to apply a fitting procedure and find a function of best fit for measured pairs of values of distance and time instead of solving the equation of motion analytically.

EQUIPMENT AND DATA ACQUISITION

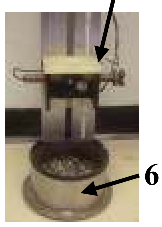


There are three setups for this experiment in MP 126.

A vertical aluminum 3-meter *track* (1) has an electromagnet (2) mounted at the top. The electromagnet is connected to a *power supply* (3). Two photogates (4) are mounted on the track; their vertical position can be changed. The photogates are connected to a *timer* (5). The timer and the power supply are placed on the table by the track.



You will suspend an object from the electromagnet above the upper photogate. A *Release* button on the power supply cuts the power to the magnet and drops the object. When the object falls through the upper *start photogate*, it starts the timer. When it falls through the lower *stop photogate*, it stops the timer. A container (6) on the floor catches the fallen object.



Clicking on the *Reset* button resets the timer. To place the object back onto the magnet, use a long wood meter stick with a small container at the top.

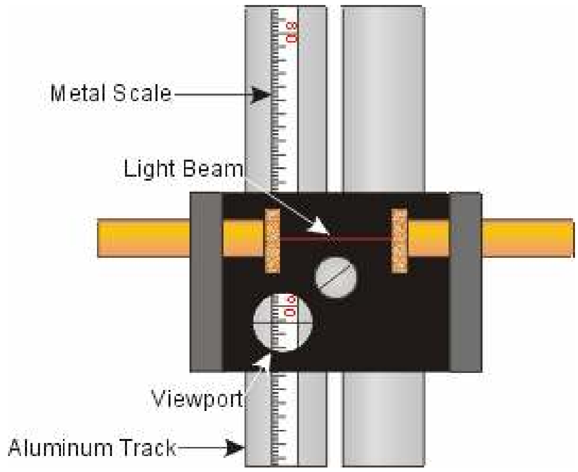
There is more than one type of object available for use with the Free Fall apparatus:

- A streamlined metal bob.
- A light plastic sphere. A small metal plug is inserted so that it may be suspended from the magnet.

In the actual experiment, the apparatus is so sensitive that repeating a measurement without moving the photogates will usually give values of the time that are slightly different. You will keep the position of the upper *start photogate* fixed throughout the experiment, so the initial speed of the object v_0 remains constant. The *start photogate* is placed about 10 centimeters below the bottom of the streamlined bob when it is suspended by the magnet.

You will move the position of the lower *stop photogate* to different positions to vary the distance s the object falls. The photogates are moved by unscrewing a large knob on the back, moving the gate to a new position, and screwing the knob back in. When screwing the knob in, make it only *finger tight*. It is easy to screw the knob in too tightly and damage the apparatus. You should place the lower *stop photogate* at 10 or so different positions. Be sure to make the range of distances s include the maximum value allowed by the apparatus. The minimum value of s should be about 10 cm.

The value of the distance s is the distance between the light beams of the two photogates. Each photogate has a viewport which allows you to see the metal scale mounted on the track underneath. Since each photogate is constructed identically, reading the positions of each gate with the viewport is equivalent to determining the positions of the light beams of the two gates.



You will need to know that the manufacturer of the metal scale quotes a *precision* of one part in 4000.

For every position of the lower gate it is a good idea to take data for both the streamlined bob and the plastic sphere at once. This will save you some time and effort.

When you are using the streamlined bob, it is almost impossible to hang it from the magnet without it oscillating. Be sure to wait until the oscillations completely stop before dropping the bob. Otherwise, at best your data will be corrupted. At worst, the bob can hit one of the photogates, possibly damaging it.

You may be wondering why we use two photogates to start and stop the timer, instead of starting the timer when we turn off the current to the magnet and stopping the timer with a single photogate. The major problem is that when we turn off the current to an electromagnet the magnetic field does not go instantaneously to zero: the Free Fall apparatus is sufficiently sensitive that the time delay between turning off the current and the object beginning to fall will mess up the data.

DATA ANALYSIS

Taking the data for this experiment is fairly straightforward. However, because of the high *precision* of the apparatus, careful analysis of your data will be necessary. Before you start the data analysis, organize your data in the most convenient way, which means actually, in a table with all measured values and their errors for two objects of different shape.

You are expected to fit your data to a number of functions to find the best fit. You may use any fitting program. In MP 126, you can use a DataStudio fitter or a MATLAB fitter (http://www.math.ufl.edu/help/matlab-tutorial/).

The simplest fit is given by a polynomial function. First function to study is given by equations (1) and (2). The goodness of this fit can show whether the drag is negligible in the experiment.

Another reasonable approximation of the solution to the equations of motion is the third-order polynomial:

$$s = s_0 + v_0 t + \frac{(g - \alpha v_0^2) t^2}{2} - \frac{\alpha v_0 (g - \alpha v_0^2) t^3}{3} \qquad (5)$$

Thus if air resistance it not negligible, the third-order polynomial can fit your data:

$$s = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \qquad (6)$$

The fitter will return values and errors for the coefficients a_0, a_1, a_2 and a_3. You may then determine the values and errors of s_0, v_0, g and α in Equation 5.

You are encouraged to find any other physically reasonable and statistically good fit for your data. Give an explanation to your choice; obtain parameters g and α with their errors.

DataStudio Fit

Click on the DataStudio icon. In this exercise, the DataStudio is not used for data acquisition; therefore choose the option “Enter Data”. Enter values from the table of measurements of your notebook into the table of DataStudio: time into the column X, and distance into the column Y. To the right-hand side you will see the plots on a graph screen.

- Choose the function to fit the data on the graph. You can try polynomial fit of different orders or any other suitable function from the list. The result will always appear with the values of constant parameters of the function, their errors and the values of *Mean Square Error* and *Root MSE*. The linear fit will also show the value of *r*. These values are given to choose the best fit.
- Explain the meaning of appeared statistical criteria of goodness of the fit.
- Compare the *RMSE* for different functions and write a conclusion about their goodness.
- Investigate the behavior of each curve in the extended range beyond the measured values of the variables and give a conclusion on the found results.
- Print out the graph that better matches experimental data. If the scale of the graph permits, show the error bars for measured values.

Excel Fit

Very similar procedure can be done using Excel. Excel permits to fit data to polynomials, exponential functions, logarithmic functions and some others.

- Click on the Excel icon on the screen of your computer and fill up the table with values of time intervals and distance traveled by the object in the free fall.
- Above the table, find a button to create a graph. Choose a *Scattered* option and follow instructions of the program to complete graphing. You can save the diagram on the same sheet with the table or create a new page.
- To see the function and parameters of the fit, find a button (*Tools*), which permits to draw a *Trendline* for the plots. You will then need to find how to set up the *Format* for the Trendline. Format has options that may be made active. You should make active the option *Display equation on the chart* and the option *Display R-squared value on the chart*. You may create a number of functions to fit data on one sheet.
- Investigate the behavior of each curve in the extended range beyond the measured values of the variables and give a conclusion on the found results.
- Print out the graph of the best fit, explain the criterion of goodness of the fit, and make a conclusion about the choice of the function. If the scale of the graph permits, show the error bars for measured values.

Other Fitting Techniques

More advanced fitting techniques, like MATLAB, not only fit the data but also take into account the errors of measured values which you could not include into the fitting with DataStudio and Excel. First, you will have to study the tutorial at http://www.math.ufl.edu/help/matlab-tutorial/ or get familiar with the contents of *Help* to know how to fit with MATLAB.

The fitter will return a value called the *chi-squared*, χ^2. It measures how closely the fitted values match the data. A smaller chi-squared means the fit is closer to the data. The value is weighted with one over the values of the errors in the data points. Thus, if the errors are large, the chi-squared is small; if the errors are small, the chi-squared is large.

The *degrees of freedom* of a fit is the number of data points minus the number of parameters to which you are fitting. Our fitter reports this number too.

The ideal value of the chi-squared is roughly equal to the number of degrees of freedom. If your chi-squared is not on the order of the number of degrees of freedom, there are three possibilities:

1. The model you are using is not appropriate. For example, if air resistance is significant and you are fitting distance versus time to only a second-order polynomial then the fit does not include the effects of air resistance. This would make the chi-squared too high. In this case you could try adding a third-order term to the polynomial.
2. You have been too optimistic in assigning errors to your data.
3. You have been too pessimistic in assigning errors to your data.

Another tool for evaluating a fit involves the *residuals* of the fit. These are the numeric values of the difference between the fitted values and the actual data values. For a good fit, the residuals should be randomly distributed around zero. If the model you are using is not appropriate for the data, the residuals will often show systematic deviations from zero.

You should look over the material **DATA FITTING TECHNIQUES** in the Lab Manual for a more complete description on the chi-squared criterion interpretation.

For this experiment, there is still another approach to determine if the effects of air resistance can be ignored. If you fit the distance versus time to a third-order polynomial, and the air resistance is negligible, the coefficient of the term for time to the third power will be zero within errors. In this case you can repeat the fit without including the third-order term. This is a general principle of least-squares fitters: fitted values that are zero within errors should almost always be excluded from the fit.

You can choose any other fitter; explain the criterion of the best fit that is used by the fitter; choose the function of the best fit and discuss the errors for the obtained parameters of the function

REFERENCES

1. R.A.Serway and J.W.Jewett, Jr. *Physics for Scientists and Engineers* 8^{th} ed. (Thomson, 2010), Vol. 1, Chapters.6 and 14.
2. R.D.Knight *Physics for Scientists and Engineer with Modern Physics* 2^{nd} ed. (Pearson, 2008), Vol. 1, Chap. 6.
3. Theodore A. Talay, *Introduction to the Aerodynamics of Flight*. (NASA, Washington, D.C., 1975).

This experiment was originally designed and fabricated by Dr. Eustace Mendis when he was here in the early 1970's. The apparatus and Guide Sheet have been revised since then by David M. Harrison, Tony Key, Ruxandra Serbanescu, Rob Smidrovskis, Joe Vise, Taek-Soon Yoon, and others.

Last updated in 2010 by Natalia Krasnopolskaia.