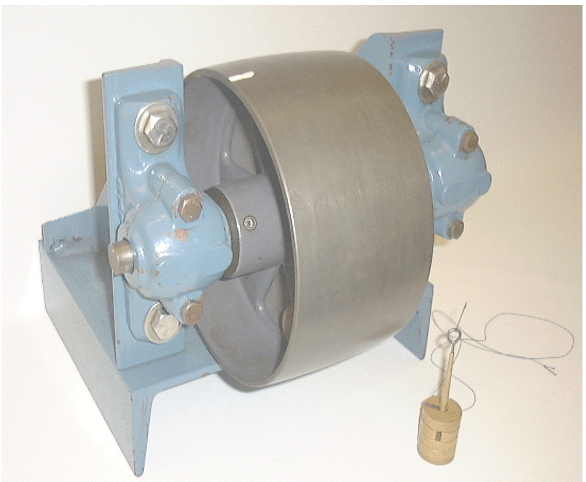
THE FLYWHEEL

REFERENCES

Most Introductory Physics texts (e.g. A. Halliday and R. Resnick, *Physics*; M.Sternheim and J.Kane, *General Physics*).

INTRODUCTION



This experiment is an introduction to some basic concepts of rotational dynamics. A fairly realistic analysis of the motion of a flywheel can be made, assuming only that the net frictional torque on a rotating flywheel is constant. In performing this experiment, you will develop understanding of:

- rotational dynamics;
- evaluation of errors in measurements that may be difficult to obtain;
- estimation of a geometrically calculated quantity using simplified models.

THEORY

The basic equations for angular motion can often be obtained simply from those for linear motion by making the following substitutions:

Linear variables	$\Rightarrow$	***Angular variables***
Force, $\vec{F}$	$\Rightarrow$	Torque, $\vec{\tau}$
Mass, m	$\Rightarrow$	Moment of Inertia, I
Velocity, $\vec{v}$	$\Rightarrow$	Angular velocity, $\vec{\omega}$
Momentum, $\vec{p}$	$\Rightarrow$	Angular Momentum, $\vec{L}$
Acceleration, $\vec{a}$	$\Rightarrow$	Angular acceleration, $\vec{\alpha}$

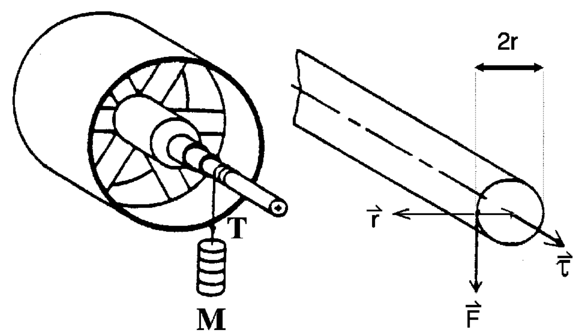
NB. The analogy needs to be treated with caution; *e.g.* I is not a constant property of the body, as is mass, since its value depends on the axis around which it is measured.

Thus Newton's Law, $\vec{F} = \dfrac{d\vec{p}}{dt} = \dfrac{d\,(m\,\vec{v})}{dt} = m\,\vec{a}$, becomes $\vec{\tau} = \dfrac{d\vec{L}}{dt} = \dfrac{d\,(I\,\vec{\omega})}{dt} = I\,\vec{\alpha}$.

(In words, the angular acceleration of a body is directly proportional to the torque applied to it, and inversely proportional to the moment of inertia of the body about the relevant axis).

The moment of inertia, **I**, is determined by imagining that the body is divided into a number of infinitesimal elements of mass $\delta \mathbf{m}_i$ each at a distance $\mathbf{r}_i$ from the axis of rotation. The moment of inertia **I** about this axis is given by the sum of all the products $(\delta \mathbf{m}_i\, r_i^2)$ calculated for each element, $\mathbf{I} = \Sigma_i (\delta \mathbf{m}_i\, r_i^2)$. If the body has a simple geometrical figure, *e.g.* a sphere, cylinder, etc., **I** can be readily calculated and results are tabulated for such bodies in the *CRC Handbook* (available in the lab) or in other first year physics texts (see **References)**.
The torque $\vec{\tau}$ of a force about an axis is given by the cross-product of the force $\vec{F}$ and the distance from the axis of rotation $\vec{r}$, *i.e.* $\vec{r} \times \vec{F}$. *(Think of opening a door - you are applying a torque whose magnitude equals the product of the force you apply and the distance from your hand to the axis of the door).*

To summarise: If a body is free to rotate about a fixed axis, then a *torque,* $\vec{\tau}$, is required to change the rotational motion of the body and an angular acceleration $\vec{\alpha}$ will result. The angular acceleration is proportional to the net torque (and exists only during the time that the torque acts) and is given by $\vec{\tau} = \mathbf{I}\vec{\alpha}$. **I** is a constant of the body known as its Moment of Inertia about the specified axis of rotation; it depends not only on mass but also the distribution of mass.



THE EXPERIMENT

In this experiment a flywheel is so mounted that torques can be applied to it by hanging a mass **M** from the free end of a string, the remainder of which is wrapped around the axle. The torque due to the weight is $\tau = \mathbf{Tr}$ where **T** is the tension in the string and **r** the radius of the axle. Because the bearings in the flywheel are not frictionless, there will be a frictional torque exerted at the bearings equal to $\vec{\tau}_f$ which will oppose the motion of the flywheel. Suppose that the string is wrapped around the axle $\mathbf{N_1}$ times and that a mass **M** is suspended from its free end and the system is released at time $\mathbf{t = 0}$. The flywheel then is given an angular acceleration, α_1, and the mass **M** accelerates downwards with an acceleration equal to $\mathbf{r}\alpha_1$.

If **T** is the tension in the string, then the net torque exerted on the flywheel is:

$$\mathbf{Tr} - \tau_f = \mathbf{I}\alpha_1$$

The net force on the mass **M** is

$$\mathbf{Mg - T = Ma = Mr}\alpha_1$$

Eliminating **T** one finds

$$\alpha_1 = \left(\frac{1}{I}\right) Mgr \left(1 - \frac{r\alpha_1}{g}\right) - \frac{\tau_f}{I} \qquad (1)$$

It may be noticed that if the frictional torque, τ_f, is constant, then the angular acceleration of the system, α_1, is also constant. With this assumption α_1 may be measured experimentally; if t_1 is the time taken for the string to unwind N_1 turns from the axle (*i.e.* the time taken for the mass **M** to drop off the axle), then the flywheel will have rotated through an angle $2\pi N_1$ radians in the time t_1 and since $\theta_1 = \alpha_1 t_1^2/2$,

$$\alpha_1 = 4\pi N_1/t_1^2 \qquad (2)$$

If the flywheel is rotating without an applied torque it will decelerate under the action of the frictional torque alone. Under this condition, equation (1) becomes

$$\alpha_2 = -\tau_f/I \qquad (3)$$

Then if it takes a time t_2 to come to rest, and in this time turns through N_2 revolutions, then the deceleration α_2 (assuming constant frictional torque) is given by

$$\alpha_2 = -4\pi N_2/t_2^2 \qquad (4)$$

If the frictional torque in the bearings is "reasonably" constant, the value of τ_f obtained by plotting (1) should be consistent with the value obtained from equation (3). The quantity **I** is by definition a constant and must be the same in equations (1) and (3). Thus, Equation (3) may be used to independently determine τ_f/I.

POINTS TO CONSIDER

In this experiment you are essentially investigating equations (1) and (3) using a massive system with little friction.

- Obtain data over as wide a range of values of applied torque as possible (torques up to 0.4 N-m are easily attainable). Also take some data at different values of **r**.

- Are you sure you counted revolutions correctly? What methods of measurement did you use?

- Consider the effect, if any of the thread you use to suspend the weights from the flywheel.

- Equation (1) suggests that fitting of α_1 *versus* $Mgr(1-r\alpha_1/g)$ should be useful. At first sight the error calculation may appear to be a little daunting; however a little thought will convince you that the error in the latter term has only two significant contributions.

  N.B. If using *Faraday* to do your fit, we strongly recommended that you use the *massage* and *recalc* facility to calculate any quantities that have long algebraic expressions; if you do the calculation of these quantities at the *fit* stage, a computer glitch often causes printing problems.

- You may find that the graphing of the data is best done on the computer; how good is the linear fit ?

- The derivation of equations (1) and (3) assumes constant frictional torque (independent of mass **M** and angular speed. Does your data justify this assumption? How would a variation in frictional torque affect your graphs?

● You might try to estimate the value of the moment of inertia, **I**, for your flywheel by considering a simplified model of its geometry and by looking-up **I** for rings, cylinders, rods, etc. The material of your flywheel is steel and the total mass of its moving parts is 6.5 kg. A rough estimate of the mass of the flywheel contained in the outer cylinder can be made by measuring the approximate volume of the different parts.

(dh -- 1983, jv -- 1988, tk -- 1995,97,98)

Preparatory Questions.

Note: We hope that the following questions will guide you in your preparation for the experiment you are about to perform. They are not meant to be particularly testing, nor do they contain any "tricks". Once you have answered them, you should be in a good position to embark on the experiment.

1. In order to calculate the angular accelerations, $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$ using Equations (2) and (4), you need a measurement of the time, **t**, it takes for **N** revolutions. Obviously the errors in **t** and **N** are closely related. Think about a simple approximate way to take account of the error in the values of the angular accelerations that you calcuate using these equations *(Hint: you do NOT need to use the formulae for propagation of errors).*

2. The flywheel can be considered to consist of three parts:

   i. a hollow outer cylinder
   ii. a solid inner cylinder
   iii. three spokes

   The contribution to the overall moment of inertia of the flywheel is dominated by the outer cylinder, whose mass is 3 kg. If the length of the outer cylinder is 10 cm, its inner and outer radii are 9.6 cm and 9.9 cm respectively, calculate an approximate value for the moment of inertia of the flywheel *(all values are approximate, and may not correspond very closely to the flywheel you use).*

3. How does the value of $\mathbf{r\alpha_1/g}$ compare with 1, given that the radius of the largest axle is approximately **r** = 3.2 cm, and that of the largest value of $\boldsymbol{\alpha_1}$ is given in the guide sheet.

4. What implication does your answer to question 3. have for the calculation of the error in the expression $\mathbf{Mgr(1-r\alpha_1/g)}$?

5. Suppose that the frictional torque increased linearly with the load on the axis; what form would you expect the plot of $\boldsymbol{\alpha_1}$ *versus* $\mathbf{Mgr(1-r\alpha_1/g)}$ to take then?