

# CHARGE AND DISCHARGE OF A CAPACITOR

## REFERENCES

- RC Circuits: Most Introductory Physics texts (e.g. A. Halliday and Resnick, *Physics* ; M. Sternheim and J. Kane, *General Physics*.)
- Electrical Instruments: This **Laboratory Manual**: *Commonly Used Instruments: The Oscilloscope and Signal Generator - Model 19*
- Stanley Wort and Richard F.M. Smith, *Student Reference Manual for Electronic Instrumentation Laboratories* (Prentice Hall 1990).  
Also see the video - *Physics Skills; How to use the Oscilloscope* - starring Dr D.M. Harrison.
- Circuit Wiring: This **Laboratory Manual**: *Circuit-Wiring Techniques: Commonly Used Instruments; the Oscilloscope*.

## OBJECTIVES

This lab will give you experience in:

- developing strategies to handle complex equipment such as oscilloscopes
- using an oscilloscope to measure a signal which varies regularly in time
- planning and wiring a simple circuit
- analyzing an exponential function obtained from data readings
- using the data analysis program on the Faraday computer

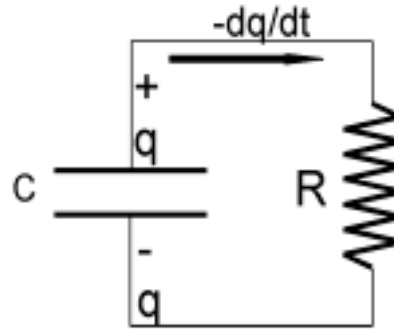
## INTRODUCTION



There are numerous natural processes in which the *rate* of change of a quantity is proportional to that quantity. One biological example is the case of population growth in which the *rate* of increase of the number of members of a species is proportional to the number present. In this case the population is said to *grow exponentially*. A radioactive example is the case in which the *rate* of loss of the number of nuclei is proportional to the number of nuclei present. The number of nuclei *decreases exponentially*. You might think

of other comparable exponential processes.

An electrical example of exponential decay is that of the discharge of a capacitor through a resistor. A capacitor stores charge, and the voltage  $V$  across the capacitor is proportional to the charge  $q$  stored, given by the relationship  $V = q/C$ , where  $C$  is called the *capacitance*. A resistor dissipates electrical energy, and the voltage  $V$  across it is proportional to the current (which is just the rate of flow of charge) through it, given by  $V = R \cdot \frac{dq}{dt}$ , where  $R$  is called



the *resistance*. When a charged capacitor is connected to a resistor, the charge flows out of the capacitor and the *rate* of loss of charge on the capacitor as the charge flows through the resistor is proportional to the voltage, and thus to the total charge present.

This can be expressed as :  $-R \frac{dq}{dt} = \frac{q}{C}$  so that  $\frac{dq}{dt} = -\frac{1}{RC} q$  (1)

which has the exponential solution  $q = q_0 e^{-\frac{t}{RC}}$  where  $q_0$  is the initial charge on the capacitor (at time  $t = 0$ ). As the voltage across the capacitor is proportional to its charge, the voltage  $V$  displays the same exponential behaviour ; divide both sides by  $C$  to obtain  $V = V_0 e^{-\frac{t}{RC}}$ .

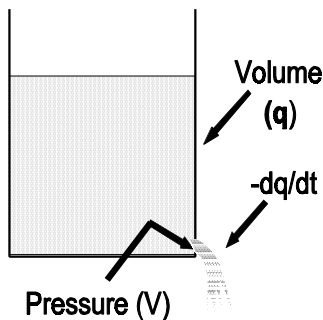


Figure 2.

This exponential decay of voltage may be likened to a water tank with a small hole in the bottom, filled with water, in which the flow or rate of loss of *volume of water* in the tank (analogous to the rate of loss of *electrical charge* from the capacitor) is proportional to the *pressure* of the water as it exerts itself on the hole (analogous to the *voltage* across the resistor). As the pressure is proportional to the height of water in the tank, which is in turn proportional to the total volume of water, we find that

the volume of water and the pressure exhibit exponential decay.

In this experiment, instead of merely discharging an already charged capacitor, you will be using an Alternating Current (AC) “square wave” voltage supply to charge the capacitor through the resistor many times per second, first in a positive direction and then in a negative direction. The charging process also exhibits the same exponential behaviour as the discharge. However this time the exponential curve approaches a constant asymptotic value rather than a zero value.

## THE EXPONENTIAL

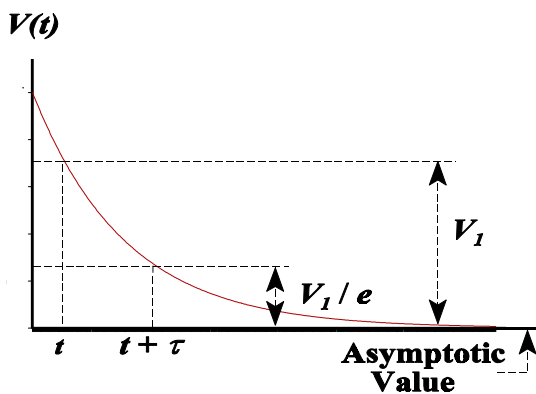
The exponential voltage function, which is derived from equation (1),

$$V(t) = V_o e^{-\frac{t}{\tau}} \quad (2)$$

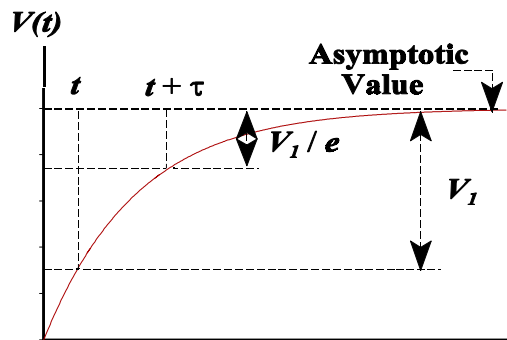
is shown in Figure 3. It has a slope (rate of change) which is proportional to the value of the function ( $V$ ) no matter where you are on the curve. Note that, in equation (2), when  $t = \tau$ ,  $V(t)$  falls to  $1/e = 0.368$  of its original value (at  $t = 0$ ).  $\tau$  is called the time constant for the exponential decay. The time to drop to  $1/e$  of a previous value is constant, no matter where on the curve you take your "initial" value.

Figure 3 illustrates the exponential decay for a *discharging* capacitor, while Figure 4 illustrates the voltage change for a *charging* capacitor. In the latter case, the voltage increases, but still approaches a constant value; this is still exponential decay, but because the voltage starts from a lower value and then rises to its asymptotic value, an additional constant term is needed in the analogue to Equation (2). The full expression in this case is  $V(t) = V_o (1 - e^{-\frac{t}{\tau}})$ .

In this experiment you will be observing repeated exponential curves; you can confirm whether the decay in voltage is exponential, and measure the time constant for that decay.



**Figure 3.**



**Figure 4.**

## THE EXPERIMENT

Connect the signal generator in series with the resistor and capacitor as shown in Figure 5.

**Note:** As with all electrical circuits, connect up the components of the circuit first, then introduce the measuring equipment (in this case the oscilloscope) only afterwards.

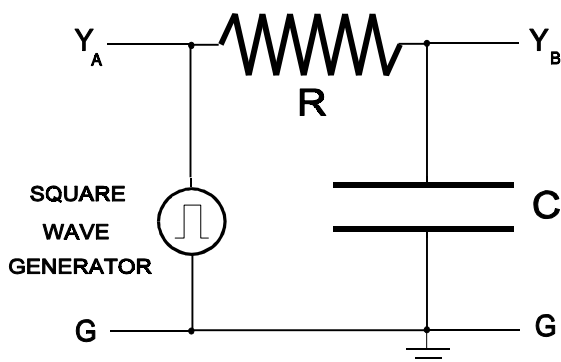


Figure 5.

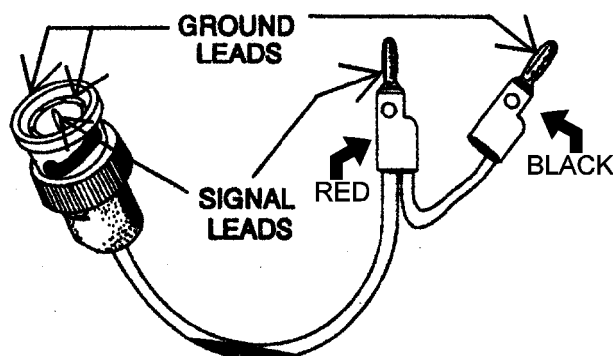


Figure 6.

Connecting the  $Y_B$  and  $Y_B$  channels of the oscilloscope as in Figure 5 will allow you to simultaneously observe the applied voltage from the signal generator (the square wave) with beam **A** of the dual trace oscilloscope, and the voltage across the capacitor,  $V_C$ , with beam **B**. The ground leads (black) of the coaxial cables, which should be connected to the ground output of the signal generator (also black), are denoted by **G**. The pairs of leads (**G**,  $Y_A$ ) and (**G**,  $Y_B$ ) represent the coaxial cables leading to the oscilloscope.

Adjust the *DC OFFSET* of the generator so that the generator output alternates between a positive voltage and zero voltage.

Use the manufacturer's values for  $C$  and  $R$  in your comparison of observation and theory. You may notice that the load placed on the signal generator by the circuit tends to distort the applied square wave; experiment with the output voltage of the signal generator to minimize this effect.

Repeat your measurements for at least two different values for  $R \times C$  and thus for the time constant.

### POINTS TO CONSIDER:

- The data you take should test whether the voltage across the discharging capacitor  $V_C$  shows exponential behaviour
- Initially choose values of frequency  $f$  which allow the capacitor to charge or discharge fully in each period. (The period of the signal from the signal generator  $T = 1/f$  should be several times the time constant  $\tau$ .) Try out a variety of values of the signal generator frequency and see what

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it does to your display.

- Obtain a “quick value” for the time constant  $\tau$ , by measuring, on the oscilloscope screen, the time required for the voltage to fall towards the asymptotic value by a factor of  $1/e$ .
- Use the oscilloscope to determine time and voltage values for particular values of  $R$  and  $C$  and record  $V_C$  as a function of  $t$ .
- If you use the data creation and analysis program on the Faraday computer to analyze your results, note that the notation used for  $\ln(x)$ , the natural logarithm of  $x$ , is **Log[x]**.
- Before dismantling the circuit, you might change the applied signal from a square wave to a sinusoidal voltage; compare the applied voltage to the voltage across the capacitor in this case. Make a qualitative record of your observations; can you give a qualitative explanation?

### COMMENTS ON THE OSCILLOSCOPE:

One of the chief pieces of learning in this experiment is finding out how to “drive” an oscilloscope. Thus, it is important that you play with the instrument, learning what the numerous controls do by trying them out. The following comments may guide you to some of the control settings:

- The **AC-0-DC** switches on the **Y<sub>A</sub>** and **Y<sub>B</sub>** sections should normally be set to **DC**. **0** is used if you want to check what the actual position of 0 volts is on the screen. **AC** alters the signal in a way that is often very useful, but it also distorts the signal. Never use **AC** except for special effects that you will learn about in second year or if you ask your demonstrator.
- There are three sets of trigger control buttons. These tell the oscilloscope when (at what point) in the signal on the screen you want the trace to start to be displayed. For the trigger source controls, use **Y<sub>A</sub>**, or **Y<sub>B</sub>**, depending on whether you want the synchronization to take place from the A or the B signal. For the trigger slope controls (**TRIG.**) choose “+” or “-”, depending on whether you want the trace on the screen to start when it is rising (positive slope) or falling (negative slope). For the trigger mode controls, choose either **AUTO** or **AC**, depending on which gives you the most stable trace. If you use **AC** mode, you may have to play with the **LEVEL** control in order to get a complete trace.
- Note that the **TIME/CM** dial affects the horizontal scaling only, and in no way moves your trace up or down. Similarly, the two **Y<sub>A</sub>** and **Y<sub>B</sub>**, **AMPL** dials affect the vertical scales only, and in no way move the trace sideways.
- When making quantitative measurements, make sure that the small knobs on the top of the main **TIME/CM** and **AMPL** controls are set to the **CAL** (“CALIBRATED”) position.

(cp-1992,1993;tk,jbv-1995)