

Elastic Properties of Solids

Exercises I – II for one weight

Exercises III and IV give the second weight

This is a rare experiment where you will get points for breaking a sample!

The recommended textbooks and other resources:

Physics for Scientists and Engineers by R.A.Serway and J.W.Jewett, Volume I, Chapter 12.

Physics for Scientists and Engineer with Modern Physics by R.D.Knight, Volume I.

http://www.pasco.com/file_downloads/product_manuals/Stress-Strain-Apparatus-without-sensors-Manual-AP-8214.pdf

INTRODUCTION

All solid objects are deformable under the applied external forces. Internal forces resist deformation. The resistance in crystals is greater than that in amorphous materials. However, the external forces can change size and shape of any solid. This change we will call *deformation*.

Stress is the quantity that is proportional to the external force causing the deformation. Stress is the external force acting on the object per unit area. *Strain* is the result of a stress. It is a measure of the degree of deformation. For sufficiently small stresses, stress is directly proportional to strain; the constant of proportionality depends on the material being deformed and on the nature of the deformation. This constant is called the *elastic modulus*:

$$\text{Elastic modulus} = \text{stress} / \text{strain}$$

We consider three types of deformation with a specific elastic modulus for each:

1. *Young's modulus* measures the resistance of a solid to a change in its length.
2. *Shear modulus* measures the resistance to motion of the planes within a solid parallel to each other.
3. *Bulk modulus* measures the resistance of solids to changes in their volume.

Consider a long uniform rod of initial length L_i suspended vertically with external force applied to its lower end and parallel to the rod. This force (action) changes the equilibrium distance among atoms in the solid and causes the internal restoring force (reaction) and elongation of the rod by the value of ΔL . In general, the increase in the length of the rod can entail a non-uniform decrease in the thickness of the rod. For small $\Delta L \ll L$, it is possible to neglect the change in the cross sectional area of the rod. In this case, the Young's modulus E is defined as

$$E = \frac{F/A}{\Delta L/L_i} = \frac{\sigma}{\epsilon}$$

where $\sigma = F/A$ is the tensile stress, F is the external force applied to the cross-sectional area A , and $\epsilon = \Delta L/L_i$ is the tensile strain. The SI unit of E in N / m^2 as stress is measured in $\text{Pa} = \text{N}/\text{m}^2$. Figure 1 shows the relationship between stress and strain in an elastic solid.

A slope of the linear *elastic region* of the graph is Young's modulus. Within the elastic region,

the solid restores its initial length and shape after the external force is removed. Elastic limit, or *yield point*, separates elastic region and *plastic region* of the graph. Deformation in the plastic region is irreversible; the object is permanently distorted and does not return to its original shape after the stress is removed. The sample breaks at the breaking point.

The objective of this lab is to find the relationship between tensile (normal) stress and tensile strain for various materials with further calculation of the Young's modulus. The Stress-Strain Apparatus stretches (and in some cases breaks) a test coupon while it measures the amount of stretch and force experienced by the test coupon. Software is used to generate a plot of stress versus strain, which allows Young's Modulus, the elastic region, the plastic region, the yield point, and the break point to be ascertained.

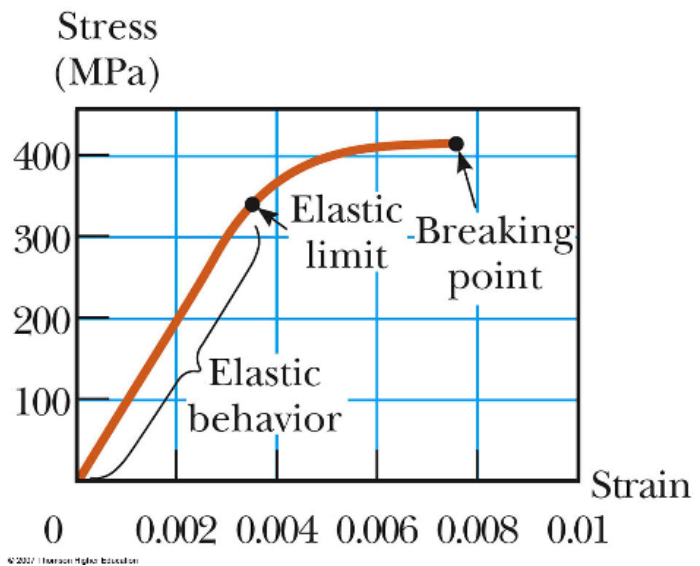


FIG.: 1. Stress vs. strain for a metal.

The Young's modulus for different materials can be found for verification here: http://www.engineeringtoolbox.com/young-modulus-d_417.html.

EXPERIMENT

The setup is shown in Fig. 2.

1. Stress-Strain Apparatus AP-8214 assembled with the Rotary Motion Sensor CI-6538 and a three-step pulley
2. Force Sensor CI-6746
3. Force sensor attachment
4. A set of three test coupons (sample stripes) of two different metals and plastic for the first one weight of the experiment and a set of four brass coupons for the second weight. The thickness of two metal coupons is 0.003 in (please, convert inches into millimeters before computation); and the thickness of the plastic coupon is 0.015 in.
5. Calibration bar
6. Tee handle with a socket
7. Caliper
8. Micrometer

The Force Sensor must be connected to Channel A of the interface, the yellow plug of the RMS must be connected to Channel 1 and the black plug - to Channel 2. PASCO ScienceWorkshop Interface and DataStudio software are used for data acquisition and processing. The DataStudio file "Stress Strain.ds" has a shortcut on the desktop of your computer.

If during the experiment the apparatus is not securely fixed on the table, use a large C-clamp to

clamp the apparatus platform to the edge of the table. The clamp is available at the Resource Center.

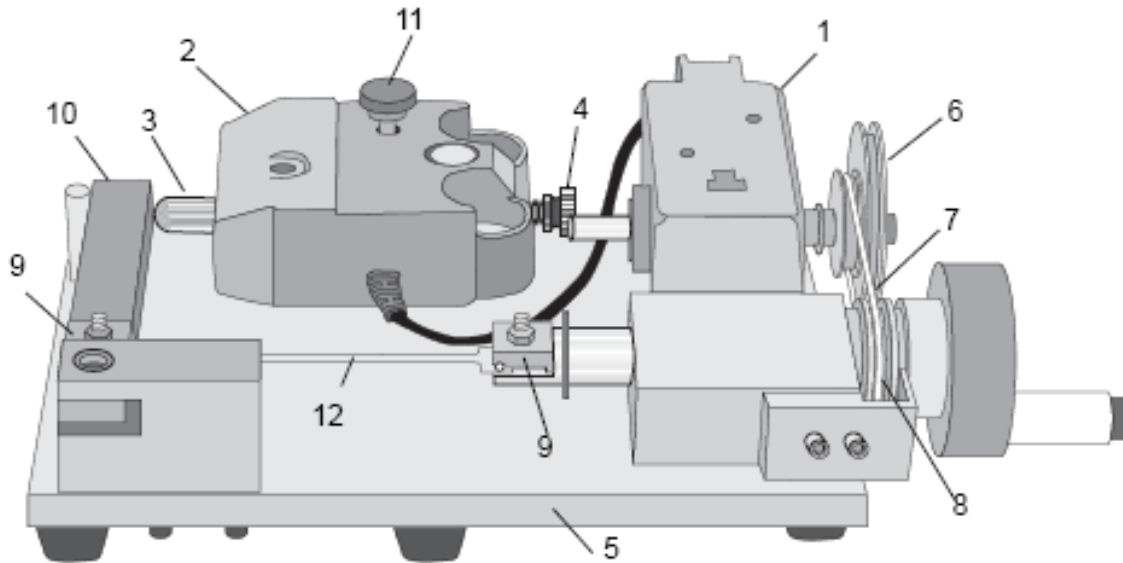
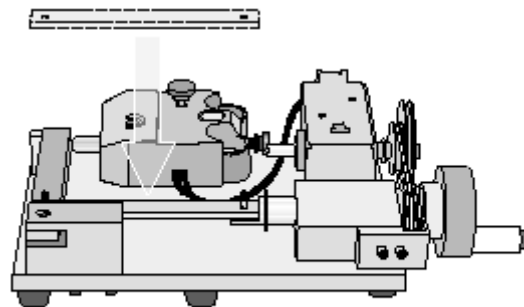


FIG.:2. Stress/Strain apparatus: 1. Rotary motion sensor
 2. Force sensor
 3. Force sensor attachment
 4. Force sensor setscrew
 5. Stress/Strain platform
 6. 3-step pulley
 7. Belt
 8. Groove
 9. Coupon clamps
 10. Lever arm
 11. Thumbscrew
 12. Coupon

Exercise I. Apparatus Calibration

During the experiment, as the crank turns, force will be applied to the test coupon, causing it to stretch. However, this force will also cause the apparatus platform and the Force Sensor to bend. The displacement registered by the RMS will be the combination of the coupon stretching and the rest of the apparatus bending.

Regardless of how much the coupon stretches, the deformation of the rest of the apparatus is constant for a given force. One can measure this deformation directly by using the calibration bar (which does not stretch significantly) in place of a coupon as force is applied. The goal is to create a plot of Displacement versus Force for the calibration bar, in which the displacement is due only to bending of the apparatus. Later, subtract this plot from a similar plot made with a coupon, in which the displacement results from both bending of the apparatus and stretching of the coupon. The result will be a plot in which the displacement is due only to stretching of the coupon.



Calibration bar setup

:

1. Install the Calibration Bar

- For each coupon clamp, remove the nut, washers, clamp top, and spring from the bolt (Fig. 3).
- Turn the crank to adjust the position of the bolts if needed and slip the calibration bar over the bolts. *Do not replace the coupon clamp parts when using the calibration bar.*

2. Place the lever arm in the starting position.

Turn the crank counter-clockwise and pull the lever arm away from the Force Sensor (Fig. 4).

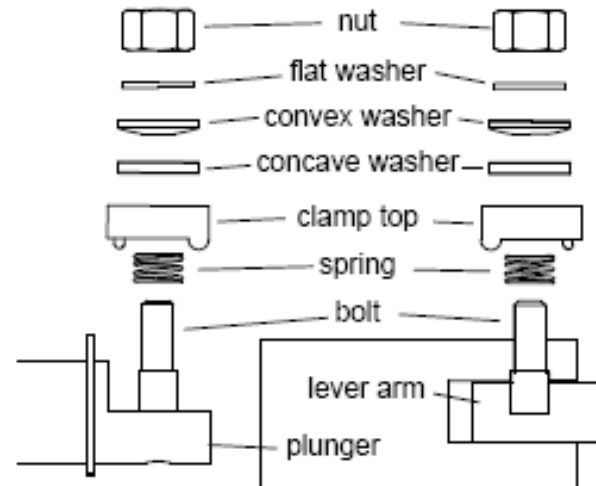


FIG.:3. Remove clamps (side view)

3. Plot Position versus Force.

Start DataStudio and open the activity file *Stress Strain.ds*. The shortcut can be found on the desktop. If you see no shortcut for this experiment, follow the steps: "Start" button -> My computer -> Disc C -> Program Files -> PASCO Scientific -> Stress Strain folder -> DataStudio files -> Stress Strain.ds.

Press the Tare button on the Force Sensor. Click the Start button. Wait until the digits of the experiment clock turn yellow. Turn the crank clockwise. Starting just before the lever arm comes into contact with the Force Sensor, turn the crank very slowly.

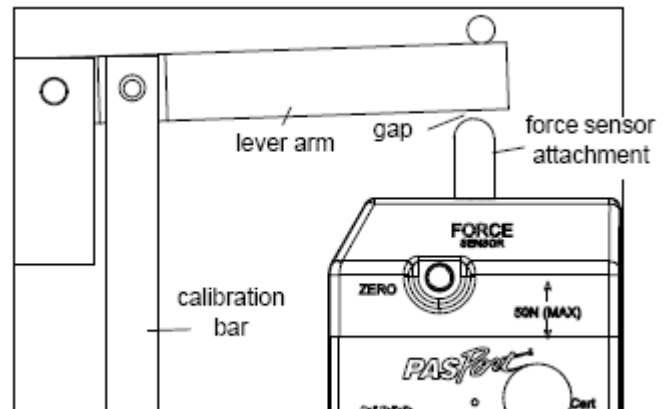


FIG.:4. Lever arm in the starting position

DataStudio will start recording when the force applied to the coupon reaches 2.5 N, or 1% of maximum (as shown in the "% Max Force" digits display). Continue to turn the crank until the force reaches 100% of maximum. At this point, DataStudio will stop recording automatically. Change the name of the data run containing the calibration data to "Cal".

*TIP:

To rename a data run, click the run name (e.g. "Run #1") where it appears in the Data List. Wait about 1 second then click it again. Enter the new name. A dialog box will appear. Select Yes. (If a window titled "Data Properties" appears, you didn't wait long enough after the first click; close that window and try again.)

Exercise II. Data Collection - Software Setup

1. **Prepare the calculation for Calibrated Position.** Click the Calculate button at the top of DataStudio. In the Calculator window, define the variables for the "Calibrated Displacement (F)" calculation:
 - Drag "Displacement(t) vs Coupon Force(t) (mm)" to "Please define variable 'Displacement'." (Fig. 5)
 - Drag "Cal" to "Please define variable 'Cal'."* (DataStudio will display a warning box stating that a "single run is selected". Click the Yes button in that box.) Click "Accept."

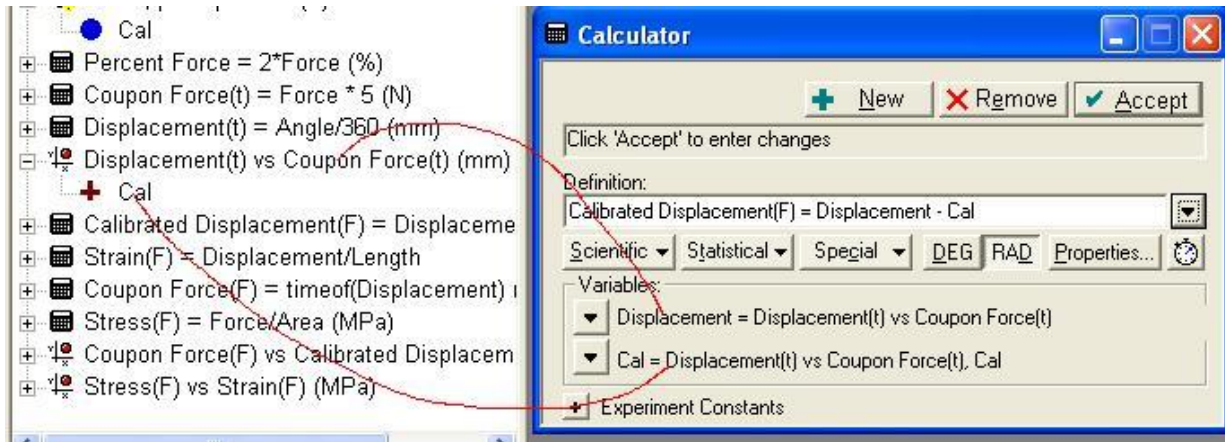


FIG.:5. Define variables in calculator

2. **Prepare the calculation for Stress.** In the Calculator window, select the defined function "Stress(F) = Force/Area". In the Variables section, enter the cross-sectional Area of the coupon *in square millimeters*. (The default setting appears). Click "Accept."
3. **Prepare the calculation for Strain.** In the Calculator window, select the defined function "Strain(F) = Displacement/Length". In the Variables section, enter the Length of the narrow part of the coupon *in millimeters*. (The default setting appears). Click "Accept."

4. Close the Calculator dialog window.

You now have the characteristic baseline curve for your particular apparatus. You can save the file and use it as the starting point for future experiments instead of repeating the calibration.

Data collection

1. Mount a coupon.

- Remove the calibration bar from the bolts. Replace the spring, clamp top, concave washer, convex washer, flat washer, and hex nut onto the bolts (Fig. 6).
- Place one end of the coupon under one of the clamp tops.
- Adjust the crank so that the opposite end of the coupon

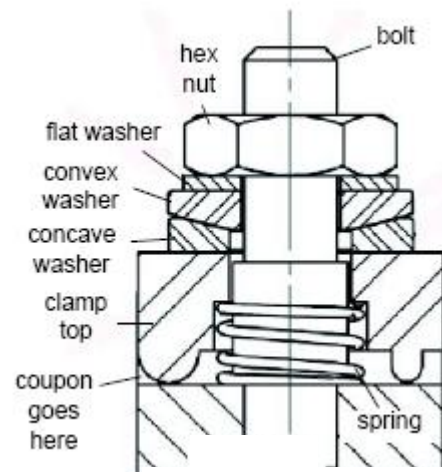


FIG.:6.

can slip easily under the other clamp (Fig. 7).

- Tighten both nuts with the tee handle with socket. With no force applied to the coupon, as little twist as possible should be visible in the coupon. *The clamps should hold the coupon tightly enough that it will not slip when force is applied. However, over-tightening the nuts will damage the bolts. Use your best judgment. If in doubt, err on the side of under-tightening.*

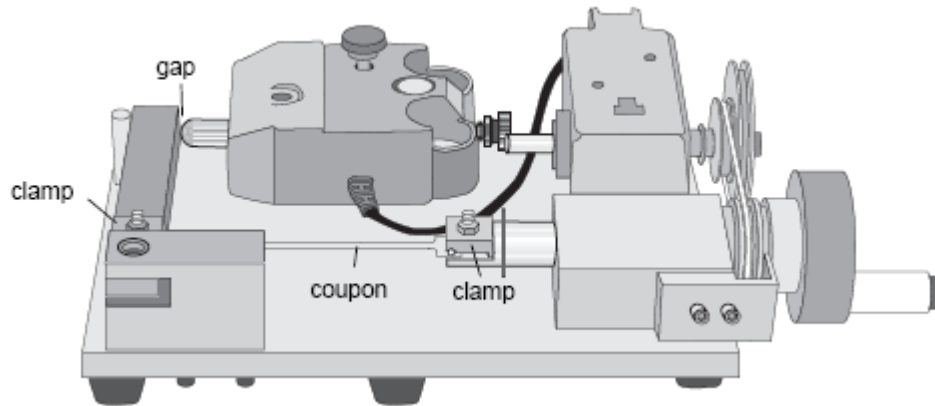


FIG.:7. Coupon installed

2. **Place the lever arm in the starting position.** Turn the crank counter-clockwise and pull the lever arm away from the Force Sensor (Fig. 4).
3. **Collect Data.**
 - Press the Tare button on the Force Sensor.
 - Click the Start button. Wait until the digits of the experiment clock turn yellow.
 - Turn the crank clockwise. Starting just before the lever arm comes into contact with the Force sensor, turn the crank very slowly. When you observe on the Stress vs Strain plot that the material has been stretched beyond the elastic region, you can begin to turn the crank faster.
 - When finished collecting data, click Stop. (If the maximum force is reached, DataStudio will stop automatically.) If the coupon breaks, it should break in the middle. If the coupon breaks near the end, it was probably twisted slightly when it was mounted, resulting in a point of higher stress where it broke.
4. **Rename the data run to identify the coupon.** Use the same method to rename the calibration data as in Exercise I.

Data Analysis

- On the Stress versus Strain graph, identify and record the elastic region, the plastic region, the yield point, and the break point (if available). To calculate Young's modulus, select a data region covering the linear, lower left-hand part of the graph. (The very first part of the plot may not be linear. This nonlinearity likely is due to the straightening of bends and twists in the coupon as force is first applied. Do not include this region in the selection.)

Apply a linear curve fit to the selected data. The slope of the line is Young's modulus in units of MPa (or MN/m² or N/mm²). Record Young's Modulus. Calculate the error for E .

- Using data for Young's modulus for different materials from http://www.engineeringtoolbox.com/young-modulus-d_417.html, and determine the substance the coupon was produced of.

Notes on the DataStudio Setup File

- For comparison of different materials, you can collect additional data runs with other coupons. Note that the Stress calculation applies only to coupons of the thickness that you entered in the Calculator window. It is easiest to compare coupons of the same thickness. However, to simultaneously display stress versus strain plots for coupons of different thicknesses, you must create a separate Stress calculation for each thickness.
- Copy the existing Stress calculation exactly (including the calculation properties), but give it a unique name (indicating the thickness for which it is designed) and enter the applicable cross-section area for the Area constant.
- When you create a new Stress calculation, note that there are two different calculations for Coupon Force- "Coupon Force(F)" and "Coupon Force(t)". Always use Coupon Force(F). The "(F)" identifies data as a function of Force, and "(t)" as a function of time. DataStudio records data as a function of time, but this experiment requires data to be recast as a function of Force. Whenever you create a new calculation or graph, be certain to use only data that is a function of Force.
- When you add a new Stress calculation to the graph, it will initially appear with time on the horizontal axis. Click the word "time" and select Strain instead.

Test two other coupons. To test another coupon, hide previous runs. From the Data button at the top of the Stress-Strain graph, deselect data runs that need to be hidden.

Return to item 2 (page 5) "Prepare the calculation for Stress." Enter the appropriate values for the calculation of Stress and Strain in the Calculator window for the new coupon. Continue with data collection.

QUESTIONS

1. Submit graphs of stress vs. strain from your data for three materials. You can sometimes guess what material is a coupon made of. Have your results for Young's modulus for three materials met your expectations?
2. Does the coupon that can withstand the greatest force also experience the greatest stress? Explain.
3. Does the coupon that can withstand the greatest force also experience the greatest strain? Explain.
4. If a coupon is stretched beyond its yield limit, but not beyond its break point, will its Young's Modulus change? Explain.
5. Describe what would happen to the Stress-Strain graph if a coupon were heated. Explain.

Exercise III. Shape and volume deformation measurement.

You will perform Exercises III and IV with brass coupons of two different thicknesses: 0.003 in and 0.005 in.

For given four coupons, measure their thickness t_i , width w_i and length L_i . Compare your results with the coupon specifications. The default value for L_i is 75 mm. Calculate the volume V_i of the piece of the coupon that will be stretched in the experiment. Record the four obtained values in your notebook.

Study the relationship between the Young's modulus and the thickness of a sample. Do it in two steps for each thickness of brass coupons.

- In first step, stretch two coupons of different thickness exactly as in the Exercises II.
- Then, stress the other two coupons below the breaking point, but still in elastic and plastic regions.

For the coupons that have not been broken, measure their average thickness t_f , average width w_f and calculate the $L_i + \Delta L$, obtained from your graph. Find the volume of the coupon been stretched V_f . Compare the results with those obtained before stretching the coupon. Make notes on what quantities have been changed due to stress and whether the shape of the coupon has also been changed.

QUESTIONS

1. Have you expected to find the Young's modulus dependent on the thickness of the coupon? Explain.
2. Compare the brass coupons of different thicknesses with respect to their Young's Modulus and yield points. Calculate the percent difference for the Young's modulus obtained for the coupons with different thickness and compare the difference to the uncertainty in E .
3. Compare the experimentally obtained value of E with the tabulated one.

Explain the results of measurements in Exercise III from the point of view of atomic structure of materials.

Exercise IV. Poisson's ratio.

In this exercise, use the coupons that were not broken in exercise III. As it is mentioned in Introduction, the deformation in one dimension causes the change in sizes in other dimensions of a solid rod. E.g. the elongation of a rod fixed between two holders can result in contraction of the rod in the perpendicular direction. Figure 8, taken from http://www.e-sunbear.com/biomech_05.html, demonstrates the width contraction accompanying the length increase.

The coupons in this experiment are cut of sheet metal and have a shape that is very different from the uniform rod or a wire. Therefore, the exact solutions for uniform rods are not applicable to the coupons. However, it is still possible to estimate the other elastic modulus

called *Poisson's ratio*. If the force is applied to the coupons as in exercises I – III, it can be considered a force that causes the *axial strain* (e.g. force F in Fig. 8), which will be denoted as ε_{axial} (positive for axial tension, negative for axial compression). This strain is accompanied by the transverse strain ε_{axial} (negative for axial tension (stretching), positive for axial compression). The Poisson's ratio is given by

$$\nu = -\frac{d\varepsilon_{trans}}{d\varepsilon_{axial}}$$

Replacing the differential values of strains by the finite changes in the values of length (the axial strain), width and thickness (the transverse strain), derive a formula for the Poisson's ratio for the coupons. Estimate the Poisson's ratio for brass coupons.

- Is the result positive or negative?
- How can you explain that the Poisson's ratio can be of both signs?
- Most materials have Poisson's ratio values ranging between 0.0 and 0.5. A perfectly incompressible material deformed elastically at small strains would have a Poisson's ratio of exactly 0.5. Why does it equal to 0.5?
- Explain the following idea of an author of the article posted here <http://www.grantadesign.com/education/sciencenote.htm> :

“You might think that the way to measure the elastic modulus of a material would be to apply a small stress (to be sure to remain in the linear-elastic region of the stress-strain curve), measure the strain, and divide one by the other. In reality, moduli measured as slopes of stress-strain curves are inaccurate, often by a factor of 2 or more, because of contributions to the strain from material creep or deflection of the test machine. Accurate moduli are measured dynamically: by exciting the natural vibrations of a beam or wire, or by measuring the velocity of longitudinal or shear sound waves in the material”.

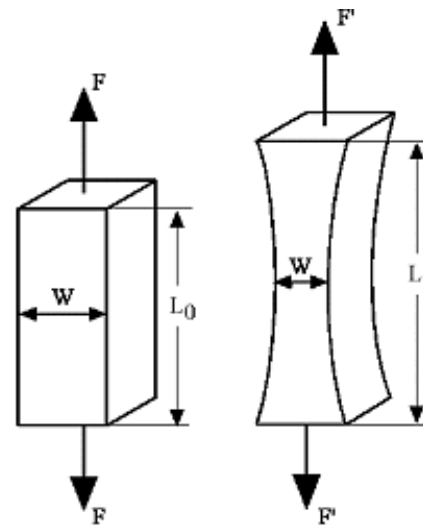


FIG.: 8. Width contraction as a result of elongation under axial stress