THE SPEED OF LIGHT
(two weights)

INTRODUCTION

This experiment is a modification of Foucault's method for the measurement of the speed of light using a low-power laser and a rotating mirror. In 1862, Foucault measured *c* = (298 000 ± 500) km/s, or an accuracy of ± 0.17% using a baseline of only 20 m. In 1929 Michelson used a development of the rotating mirror arrangement over a baseline of 22 miles between Mt. Wilson and Mt. San Antonio in California to determine c = (299 796 ± 4) km/s. Although the technique you use is similar, you can probably only expect to achieve accuracies of ± 5%. You will find as did Foucault, that there is a limit to the accuracy achievable in a technique dependent on the measurement of the displacement of a light beam. It is interesting, however, considering the magnitude of the velocity being measured, that one can quite easily measure *c* in one or two laboratory periods.

You will be among several students to assemble this sophisticated experiment at its new location, where it has never been set up before. Actually, you will be one of the creators of this experiment at the Department of Physics University of Toronto. Assembling the setup you will have to apply your experimentalist's skills along with your knowledge of Physics laws and display talent of an engineer.

The other challenge of this experiment is the solving of the problem of how to set-up an apparatus *systematically*. If you work out your system of alignment carefully, you can perform the experiment easily. However, if you do not proceed systematically, you could spend a couple of lab periods without obtaining any results.

THEORY OF THE METHOD

In principle, the apparatus is that of Figure 1. (Our actual set-up can achieve a longer flight path for the light by inserting the extra mirror(s) between $\mathbf{M_o}$ and **L** so that the light beam can double back in the room.)

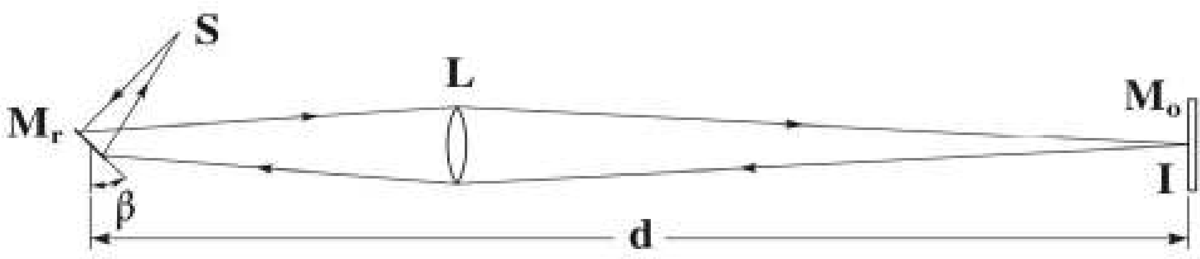

Fig.1. Principle scheme of the

Light from a source **S** falls onto the rotating mirror $\mathbf{M_r}$ and is focussed by the lens **L** to form an image **I** on the surface of the mirror $\mathbf{M_o}$. The light from this image, reflected by $\mathbf{M_o}$, will form a second image exactly coincident with the original object **S**, since **I** and S are conjugate points of the lens **L**. Note that this is true for all rays that pass through **L** and reach $\mathbf{M_o}$, even though these rays will not generally retrace their paths exactly (as they did in the original Foucault method), because of the focussing properties of **L.**

An angle β is defined as the instantaneous angle between the plane of the rotating mirror and some arbitrary direction. It is important to note that the position of the final image on return to the source location **S** is independent of the angle β of rotating mirror $\mathbf{M_r}$, although the position of the image **I** on the face of mirror $\mathbf{M_o}$ will vary with angle β. (Again, note that this is true only if the rays do pass through lens **L** and do strike the surface of $\mathbf{M_o}$ - otherwise the light just won't make it back to the location of **S**.)

The above considerations are true for $\mathbf{M_r}$ stationary at any reasonable angle β or rotating at some low value of angular velocity $\omega = \frac{d\beta}{dt}$. However, if ω is sufficiently large so that the mirror $\mathbf{M_r}$ has rotated through an angle $\delta\beta$ in the time the light takes to travel from $\mathbf{M_r}$ to $\mathbf{M_o}$ and back, the final image will be displaced sideways from the original source position **S**.

Referring to Figure 2, if the rotating mirror is initially at position **1** then a line from **S** to **P** is perpendicular to $\mathbf{M_r}$ and the source **S** produces an image in $\mathbf{M_r}$ at **P**. The light, after reflection from $\mathbf{M_o}$ heads to **P** on its return and forms an image at **S**. But if $\mathbf{M_r}$ is now at position **2**, the light is reflected at a larger angle to form an image at **S'**. If the mirror has rotated through an angle $\delta\beta$, using the laws of reflection, you should be able to show that the angle subtended at the mirror $\mathbf{M_r}$ by **S** and **S'** is $2 \cdot \delta\beta$. Thus, the displacement **x** of the final image, due to change in mirror angle from when the light leaves $\mathbf{M_r}$ to when it returns is just $\mathbf{x} = l \cdot 2\delta\beta$, assuming $\delta\beta$ to be small, and where l is the distance from $\mathbf{M_r}$ to **S**.

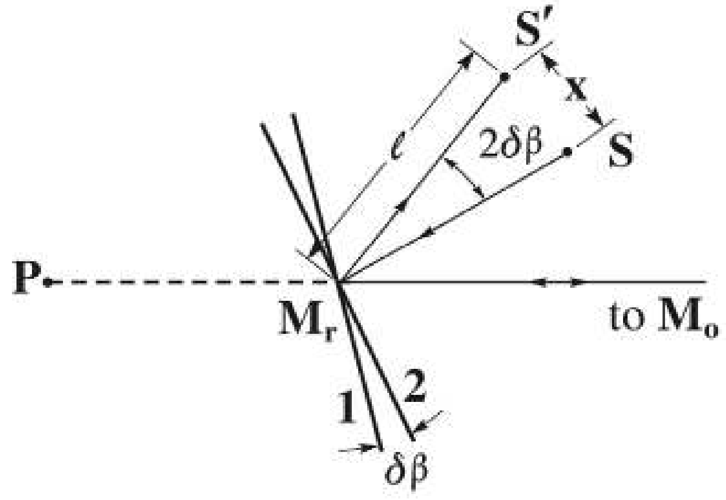

Fig.2. Formation of image in the rotating mirror.

If the optical path length between $\mathbf{M_r}$ and $\mathbf{M_o}$ is d, then the light, travelling at a speed c, takes a time $\delta t = 2d/c$ between reflections at $\mathbf{M_r}$, so that if the mirror is rotating at an angular velocity ω, then $\delta\beta = \omega\,\delta t = 2\omega d/c$; and thus

$$x = 4\frac{ld}{c}\omega \qquad (1).$$

The determination of c thus reduces to the measurement of two fixed distances, a variable angular velocity, and a dependent displacement of an imaged light beam.

SPEED OF LIGHT

EXPERIMENT

The scheme of our method to measure the speed of light is shown in Fig.3.

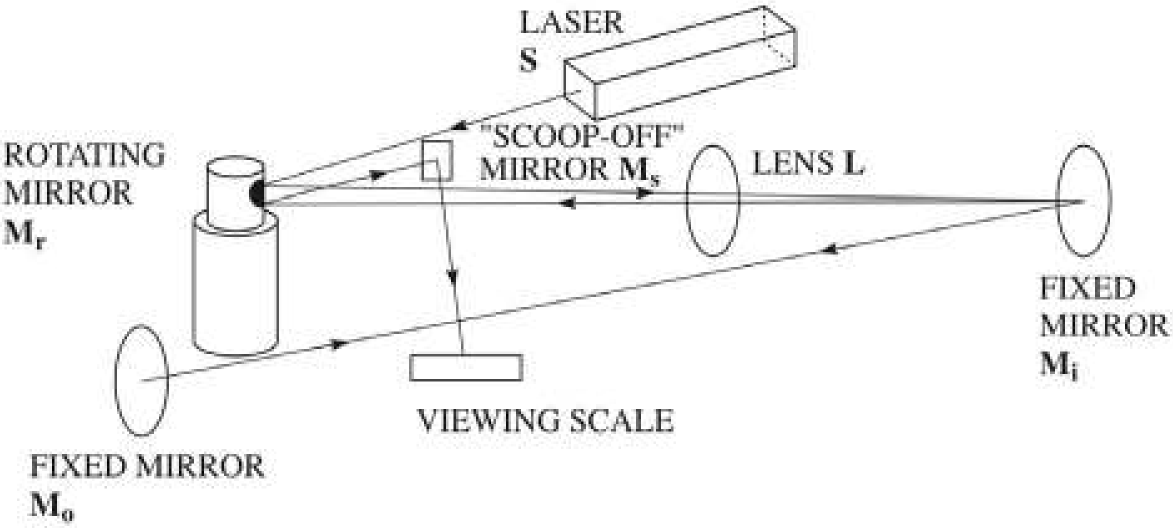

Fig.3. Arrangement of the main parts of the device

Although the method requires merely a small intense light source (such as a small intense focused light and a slit), we have provided you with a laser light source. Lasers have the advantage of lots of intensity, all directed in the direction you want. However they have the disadvantage of having too small an angular spread for this experiment so that if the rotating mirror gets even slightly misaligned (as can happen at high rotational speeds if the motor shaft is at all out of balance) then the beam can end up completely missing mirror $\mathbf{M_0}$.

Note that the mirrors used in this experiment are aluminized-front-surface mirrors. They do not have a protective overcoating. Great care should be taken not to touch the mirrored surface nor should any attempt be made to clean them, otherwise damage to the mirror will surely result.

> **CAUTION: If viewed directly, the beam from even a low power laser can cause permanent damage to your vision. Never look directly into the beam without a diffusing screen between you and the laser.**

As mentioned in the introduction, most of the difficulties in this experiment lie in the process of arriving at a good optical arrangement.

The rotating mirror $\mathbf{M_r}$ and the fixed mirrors $\mathbf{M_0}$ and $\mathbf{M_i}$ are placed to give the maximum path length possible given the finite size of the space. The rotating mirror $\mathbf{M_r}$ can be rotated by hand by inserting the removable knob into the top of the motor assembly. As a safety feature, the removable knob is tethered to an elastic cord. Removing the knob before starting the motor prevents accidental damage to the motor shaft.

Preliminary Beam Alignment

Hints:

- You may do fine adjustment of mirrors by gently rotating their base stands and by changing their tilt by using their base stand adjusting screws.
- In directing the beam, it is often easier to rotate the appropriate mirror rather than run long distances to translate another mirror sideways or vertically.
- Even one person working alone on the experiment should be able to line-up the beam with at most two walking trips up and down the full light path.
- It is useful to start your line-up by adjusting all mirrors and the laser source to be the same height.

With lens **L** removed from the light path, use the following steps as a guide to form the path

$$\mathbf{S} \Rightarrow \mathbf{M_r} \Rightarrow \mathbf{M_i} \Rightarrow \mathbf{M_o} \Rightarrow \mathbf{M_i} \Rightarrow \mathbf{M_r} \Rightarrow \mathbf{S}$$

1. Adjust the laser **S** and its stand so that the beam strikes the centre of $\mathbf{M_r}$ and reflects back to **S**.
2. Adjust $\mathbf{M_r}$ and the laser so that the beam rotates in a horizontal plane as $\mathbf{M_r}$ is rotated. When making corrections to the alignment, make only partial corrections with either $\mathbf{M_r}$ or **S**. Note that when $\mathbf{M_r}$ is adjusted, **S** must also be simultaneously adjusted so that the beam still strikes the centre of $\mathbf{M_r}$ and still reflects back to **S**. This procedure may require several iterations.
3. Check that the centre of $\mathbf{M_o}$ is at the same height as the centre of $\mathbf{M_r}$. If it is not, make the necessary adjustments.
4. Adjust $\mathbf{M_i}$ so that the beam hits its centre and use the adjustment screws so that the reflected beam strikes the centre of $\mathbf{M_o}$.
5. Adjust $\mathbf{M_o}$ so that the reflected beam hits the centre of $\mathbf{M_i}$ and, even though it is large, the centre of the beam reflecting from $\mathbf{M_i}$ again strikes $\mathbf{M_r}$. You should see a large red spot which is a rectangular image of $\mathbf{M_r}$ back at the laser source **S**.

Introduction of the Lens L

6. Calculate the position for lens **L** to make **S** and $\mathbf{M_o}$ optically conjugate (i.e., **L** causes an object at **S** to form an image at $\mathbf{M_o}$). Use the thin lens formula $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ where s and s' are the object and image distances respectively. You will need to know the total distance from **S** to $\mathbf{M_o}$. The focal length f, of **L**, is given in millimetres on the lens holder. Although the correct answer requires the solution of a quadratic equation, you should be able to estimate the correct answer in your head.
7. Place **L** in its calculated location and adjust its vertical and transverse position so that the beam now traverses the path

$$\mathbf{S} \Rightarrow \mathbf{M_r} \Rightarrow \mathbf{L} \Rightarrow \mathbf{M_i} \Rightarrow \mathbf{M_o} \Rightarrow \mathbf{M_i} \Rightarrow \mathbf{L} \Rightarrow \mathbf{M_r} \Rightarrow \mathbf{S}$$

Introduction of the "Scoop Off" Mirror M_S

By this point in the set-up, you have the beam returning to the laser source **S**. If you were to now run the motor to rotate $\mathbf{M_r}$ you would not be able to make measurements since this would require that your eye be inside the laser. Thus you must now insert the "scoop-off" small mirror $\mathbf{M_s}$ between the laser **S** and $\mathbf{M_r}$, angled in such a way that the returning light beam gets reflected on to your observing scale rather than returning to the laser **S**.

Since the beam is very intense, use the coloured filter provided to reduce the brightness when viewing the beam through the magnifying eyepiece. Note that the smallest divisions on the viewing scale are 0.5 mm. The coloured filter is to be removed later when $\mathbf{M_r}$ is in motion.

8. The "scoop-off" is achieved by first making a very slight adjustment of the laser stand so that the beam strikes $\mathbf{M_r}$ near its top. Blocking $\mathbf{M_o}$ will make this adjustment easier.

9. Probably the returning beam will be below $\mathbf{M_r}$ (there may be multiple reflected beams but the returning beam will be the brightest). Make a very slight adjustment of the tilt of $\mathbf{M_o}$ so that the returning beam strikes $\mathbf{M_r}$ near its bottom. Use a piece of paper near $\mathbf{M_r}$ to view the separation between the outgoing and returning beams.

10. Now $\mathbf{M_s}$ may be placed near $\mathbf{M_r}$ such that it intercepts only the beam travelling from $\mathbf{M_r}$ headed towards **S,** but does not intercept any of the three other light beams arriving at or leaving $\mathbf{M_r}$. Adjust the angle of $\mathbf{M_s}$ so that the outgoing beam hits the viewing scale.

11. Position the viewing scale at such a distance that the light spot is a minimum size. Although you should not move $\mathbf{M_r}$ now, it should be pointed out that at this position the spot will not move from side to side when **Mr** is rotated through small angles. Later when $\mathbf{M_r}$ is rotating, the beam hits the scale for only a small range of angles and this is why you see a circular spot and not a flattened ellipse.

12. Turn on the autotransformer and set the motor rotating at a low speed. Remove the coloured filter from the beam path. Using the eyepiece, fine tune the position of the scale so that the spot on the scale is round and small in size.

Measurements

13. Note that the definition of l now becomes the distance from $\mathbf{M_r}$ to $\mathbf{M_s}$ to the viewing scale. When the viewing scale is placed at the position where the spot size is a minimum, theory says that the light path distance from $\mathbf{M_r}$ to $\mathbf{M_s}$ to the viewing scale is the same as the distance from **S** to $\mathbf{M_r}$. Measure l.

14. Measure **d**.

15. The time of rotation of the mirror and hence ω can be measured using the photodiode. Connect the photodiode to **CH1** of the oscilloscope. Press **CH1 Menu** and set:
    Coupling - DC
    BW Limit - ON

VOLTS/DIV - Coarse
Probe – 1X
INVERT - OFF

With $\mathbf{M_r}$ stationary and with the detector in a position such that no part of the beam is intercepted by the motor housing, adjust the photodiode for maximum signal which should be greater than 5 V.

16. To determine the frequency when $\mathbf{M_r}$ is rotating, press **MEASURE**, choose **Type** and set **CH1** to **Freq**. Adjust **VOLT/DIV**, **SEC/DIV** and **TRIGGER LEVEL** as the motor speed changes to keep at least two peaks on the screen and the frequency measurement valid.

   Without touching the mirror surfaces, when you determine ω, be sure to check whether the rotating mirror is single or double sided as this will affect your calculations.

17. Measure x as a function of ω over as large a range of ω as is practical. Take some of your data with ω increasing and some with ω decreasing and keep track of the order in which data is recorded. At high frequencies the photodiode may have to be adjusted for maximum signal.

You could run your experiment using the viewing scale. However, for greater precision you may substitute the scale and magnifying eyepiece assembly for the viewing scale. Also you may insert a slit directly in from of the laser. Choose the slit width to optimize clarity, sharpness and intensity of image as seen in the viewing eyepiece.

DATA INTERPRETATION

The form of equation (1) immediately suggests a graphical way of interpreting the data. Note that x in equation (1) represents the displacement of the final light spot from its position when the mirror is not rotating. The measuring scale will probably be arbitrarily positioned with its zero not corresponding to the position of the spot for a stationary $\mathbf{M_r}$**.** Thus equation (1) is better written as:

$$x = \frac{4ld}{c}\omega + x_0 \qquad (2)$$

Do not try to determine x_o with the mirror stationary as the brightness of the spot is so much greater than when the mirror is rotating that the data taken under the two intensity conditions is not comparable.

This experiment is ideally suited to plotting your data while you take it and then to apply the linear fit of DataStudio to the function in equation (2). Make sure that you have sufficient number of points to obtain a useful result. Take some of your data with ω increasing, and some with ω decreasing and keep track, on your graph, of each set of data. Be sure to take points for as large a range of values of ω as is practical.

Opening the DataStudio, choose the option “Enter Data” and fill up the table. To the right-hand side you will see the plots on a graph screen. Calculate c from the slope of the line and the error of c using the function (2) and the rules of propagation of the error.

Modified by John Pitre in 2006
Last updated by Natalia Krasnopolskaia in 2008